**PROBLEM:**
A linear time-invariant discrete-time system is described by the difference equation

\[ y[n] = 3x[n] - 2x[n - 1] + 2x[n - 2] - 3x[n - 4]. \]

(a) Draw a block diagram that represents this system in terms of unit-delay elements, coefficient multipliers, and adders.

(b) Determine the impulse response \( h[n] \) for this system. Express your answer as a sum of scaled and shifted unit impulse sequences.

(c) Use convolution to determine the output due to the input

\[ x[n] = \delta[n] + 2\delta[n - 1] + \delta[n - 2] \]

Plot the output sequence \( y[n] \) for \(-3 \leq n \leq 10\).

(d) Now consider another LTI system whose impulse response is

\[ h_d[n] = \delta[n] + 2\delta[n - 1] + \delta[n - 2]. \]

Use convolution again to determine \( y_d[n] = x_d[n] \ast h_d[n] \), the output of this system when the input is

\[ x_d[n] = 3\delta[n] - 2\delta[n - 1] + 2\delta[n - 2] - 3\delta[n - 4]. \]

How does your answer compare to the answer in part (c)? This example illustrates the general commutative property of convolution; i.e., \( x[n] \ast h[n] = h[n] \ast x[n] \).
Part A

\[ x[n] \rightarrow \text{Multiplier} \rightarrow D \rightarrow \text{Adder} \rightarrow y[n] \]

Unit Delay Elements

Part B

Plugging \( x[n] = \delta[n] \) into the difference equations yields the output

\[ h[n] = 3\delta[n] - 2\delta[n-1] + 2\delta[n-2] - 3\delta[n-4] \]

Part C

\[ y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] \]

\[ = \sum_{k=0}^{2} x[k] h[n-k] \quad \text{(since } x[k] = 0 \text{ for } k < 0 \text{ and } k > 2) \]


\[ = 3 \delta[n] - 2 \delta[n-1] + 2 \delta[n-2] - 3 \delta[n-4] + 6 \delta[n-1] - 4 \delta[n-2] + 4 \delta[n-3] - 6 \delta[n-5] + 3 \delta[n-2] - 2 \delta[n-3] + 2 \delta[n-4] - 3 \delta[n-6] \]

\[ = \frac{3 \delta[n] + 4 \delta[n-1] + \delta[n-2] + 2 \delta[n-3] - \delta[n-4] - 6 \delta[n-5] - 3 \delta[n-6]}{2} \]
Part D

\[ y_d[n] = x_d[n] * h_d[n] = \sum_{k=-\infty}^{\infty} x_d[k] h_d[n-k] \]

\[ = \sum_{k=0}^{4} x_d[k] h_d[n-k] \quad \text{(since } x_d[k] = 0 \text{ for } k < 0 \text{ and } k > 4) \]


\[ = 3 h_d[n] - 2 h_d[n-1] + 2 h_d[n-2] - 3 h_d[n-4] \]

\[ = 3(\delta[n] + 2 \delta[n-1] + \delta[n-2]) - 2(\delta[n-1] + 2 \delta[n-2] + \delta[n-3]) + \\
2(\delta[n-2] + 2 \delta[n-3] + \delta[n-4]) - 3(\delta[n-4] + 2 \delta[n-5] + \delta[n-6]) \]

\[ = 3 \delta[n] + 4 \delta[n-1] + \delta[n-2] + 2 \delta[n-3] - \delta[n-4] - 6 \delta[n-5] - 3 \delta[n-6] \]

\[ = \text{ Same answer as part (c).} \]