**PROBLEM:**

For a particular linear time-invariant system, when the input is

\[ x_1[n] = 4u[n] = \begin{cases} 
0 & n < 0 \\
4 & n \geq 0 
\end{cases} \]

the corresponding output is

\[ y_1[n] = \delta[n] + 2\delta[n - 1] + 3\delta[n - 2] + 4u[n - 3] = \begin{cases} 
0 & n < 0 \\
1 & n = 0 \\
2 & n = 1 \\
3 & n = 2 \\
4 & n \geq 3 
\end{cases} \]

(a) Using the concepts of linearity and time-invariance, determine the impulse response of the system.

(b) The system is an FIR filter—determine the filter coefficients and the length of the filter.

(c) State a general procedure for deriving the impulse response of a LTI system from a measurement of its step response, i.e., if \( s[n] \) is the step response of a LTI system, what simple operations can be done to \( s[n] \) to produce the impulse response \( h[n] \).

(d) Using the concepts of linearity and time-invariance, determine the output signal when the input signal is \( x_2[n] = 7u[n - 1] - 7u[n - 4] \). Give your answer as a formula expressing \( y_2[n] \) in terms of known sequences or as an equation for each value of \( y_2[n] \) for \( -\infty < n < \infty \).
(a) \( x_1[n] \rightarrow y_1[n] \) is a known input-output pair. We need to express \( \delta[n] \) in terms of \( x_1[n] \) which means we are allowed to shift and scale \( x_1[n] \) to create \( \delta[n] \). In other words, can we find \( \alpha_1, n_1, \alpha_2 \) and \( n_2 \) so that
\[
\delta[n] = \alpha_1 x_1[n-n_1] + \alpha_2 x_1[n-n_2]
\]
Maybe it is easiest to see from plots. Make a plot of \( x_1[n] \)

\[ x_1[n] \]

\[ \begin{array}{cccccc}
4 & 0 & 0 & 0 & 0 & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & 1 & 2 & 3 & 4 & 5 \\
\end{array} \]

Make a plot \( x_1[n] \) shifted by one (delay).

\[ x_1[n-1] \]

\[ \begin{array}{cccccc}
4 & 0 & 0 & 0 & 0 & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
1 & 2 & 3 & 4 & \cdots & \cdots \\
\end{array} \]

If we want to combine these two plots to get \( \delta[n] \)

\[ \begin{array}{cccccc}
1 & 0 & 0 & 0 & 0 & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & 1 & 2 & 3 & 4 \\
\end{array} \]

Then, clearly we need to subtract, and divide by 4.

Thus \( \delta[n] = \frac{1}{4} x_1[n] - \frac{1}{4} x_1[n-1] \)
Prob (cont)

(a, cont) Now we can use linearity

If \( s[n] = \frac{1}{4} x_1[n] - \frac{1}{4} x_1[n-1] \)
then \( h[n] = \frac{1}{4} y_1[n] - \frac{1}{4} y_1[n-1] \)

**MAKE A TABLE TO COMPUTE h[n].**

<table>
<thead>
<tr>
<th>n ( n &lt; 0 )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>( n \ge 9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>y_1[n]</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>\cdots</td>
</tr>
<tr>
<td>y_1[n-1]</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>\cdots</td>
</tr>
<tr>
<td>h[n]</td>
<td>0</td>
<td>\frac{1}{4}</td>
<td>\frac{1}{4}</td>
<td>\frac{1}{4}</td>
<td>\frac{1}{4}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>\cdots</td>
</tr>
</tbody>
</table>

\( \sum h[n] = \frac{1}{4}(3) - \frac{1}{4}(2) = \frac{3}{4} - \frac{1}{2} = \frac{1}{4} \)

(b) For an FIR filter the impulse response will "read out" the coefficients.

\( \Rightarrow \{ b_k \} = \{ \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \} \) \( L = 4 \) is FILTER LENGTH

(c) Since \( s[n] = u[n] - u[n-1] \), if we have the step response \( s[n] \), the impulse response \( h[n] \) can be constructed via: \( h[n] = s[n] - s[n-1] \).

(d) Write \( x_2[n] \) in terms of \( x_1[n] \)

\( x_2[n] = \frac{1}{4} x_1[n-1] - \frac{1}{4} x_1[n-4] \).

\( \Rightarrow y_2[n] = \frac{1}{4} y_1[n-1] - \frac{1}{4} y_1[n-4] \) \( \text{USING L.T.I.} \)

**MAKE A TABLE**

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<th>7</th>
<th>8</th>
<th>( n \ge 9 )</th>
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</thead>
<tbody>
<tr>
<td>y_1[n-1]</td>
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<td>0</td>
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<td>\cdots</td>
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<tr>
<td>y_2[n]</td>
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<td>0</td>
<td>\frac{1}{4}</td>
<td>\frac{1}{2}</td>
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