PROBLEM:
An FIR filter is described by the difference equation:

\[ y[n] = 3x[n] + 2x[n - 3] - 3x[n - 5] \]

(a) Find its impulse response \( h[n] \) and plot versus \( n \).

(b) Let \( x[n] \) be the complex exponential

\[ x[n] = 3e^{j(0.4\pi n - \pi/2)} \quad \text{for all } n \]

Then it is possible to express the output \( y[n] \) in the form

\[ y[n] = Ae^{j(\omega_0 n + \phi)} \]

Determine the numerical values of \( A, \phi \) and \( \omega_0 \).
(a) \[ y[n] = 3x[n] + 2x[n-3] - 3x[n-5] \]
\[ b_0 = 3 \quad b_3 = 2 \quad b_5 = -3 \]
Let \( x[n] = \delta[n] \)
\[ h[n] = 3 \delta[n] + 2 \delta[n-3] - 3 \delta[n-5] \]

- \( h[n] \) will "read out" the filter coeffs.

(b) \( x[n] = 3e^{-jn/2}e^{j0.4\pi n} \)
\[ y[n] = \left( H(\hat{\omega}) \bigg|_{\hat{\omega} = 0.4\pi} \right) x[n] \]
\[ H(\hat{\omega}) = 3 + 2e^{j3\hat{\omega}} - 3e^{j5\hat{\omega}} \]
\[ = 3 + 2e^{j1.2\pi} - 3e^{j2\pi} = 3 + 2e^{j1.2\pi} - 3 = 2e^{j\pi} \]
\[ \Rightarrow y[n] = (2e^{j1.2\pi}) 3e^{-jn/2}e^{j0.4\pi n} = 6e^{-jn/2}e^{j0.4\pi n} \]
\[ A = 6, \phi = -1.7\pi, \hat{\omega}_0 = 0.4\pi \]
We get the same result if we just plug into the difference equation:
\[ y[n] = 9e^{-jn/2}e^{j0.4\pi n} + 6e^{-jn/2}e^{j0.4\pi(n-3)} - 9e^{-jn/2}e^{j0.4\pi(n-5)} \]
\[ = 3e^{-jn/2}e^{j0.4\pi n}(3 + 2e^{j1.2\pi} - 3e^{j2\pi}) \]
\[ H(\hat{\omega}) \text{ at } \hat{\omega} = \hat{\omega}_0 = 0.4\pi \]