PROBLEM:
Evaluate the “running” average:

\[
y[n] = \frac{1}{L} \sum_{k=0}^{L-1} x[n - k]
\]

when the input signal is a geometric sequence. As before, compute the numerical values of \(y[n]\) over the range \(-5 \leq n \leq 10\); let \(L = 4\) and \(a = 0.8\). Then derive a general formula for \(y[n]\) that will apply for any value of the parameters \(L\) and \(a\), when \(n \geq 0\).

\[
x[n] = \begin{cases} 
0 & \text{for } n < 0 \\
a^n & \text{for } n \geq 0 
\end{cases}
\]

If you cannot visualize the output signal, use MATLAB to create a plot of the output for \(a = 0.9\) and \(a = 0.8\) over the range \(0 \leq n \leq 20\). Take \(L\) to be equal to 7.


SOLUTION
\[ x[n] = a^n u[n] \text{ is input} \]

4-Point Running Avg.

\[
\begin{array}{c|c|c|c|c}
 n & x[n] \\
\hline
 0 & 1 \\
 1 & 0.8 \\
 2 & 0.64 \\
 3 & 0.512 \\
 4 & 0.4096 \\
 5 & 0.328 \\
\end{array}
\]

\[
y[0] = \frac{1}{4} (1 + 0.8 + 0 + 0) = \frac{1}{4} = 0.25 \\
y[1] = \frac{1}{4} (0.8 + 1 + 0 + 0) = 0.45 \\
y[2] = \frac{1}{4} (0.8 + 1 + 0 + 0) = \frac{2.44}{4} = 0.61 \\
y[3] = \frac{1}{4} (0.512 + 0.64 + 1 + 0) = 0.738 \\
y[4] = \frac{1}{4} (0.4096 + 0.512 + 0.64 + 0.8) = 0.5904
\]

**Problem values for the 4-point running average**

\[
y = \\
\text{Columns 1 through 7} \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.2500 \quad 0.4500 \\
\text{Columns 8 through 14} \\
0.6100 \quad 0.7380 \quad 0.5904 \quad 0.4723 \quad 0.3779 \quad 0.3023 \quad 0.2418 \\
\text{Columns 15 through 21} \\
0.1935 \quad 0.1548 \quad 0.1238 \quad 0.0991 \quad 0.0792 \quad 0.0634 \quad 0.0507
\]

**Problem values for the 7-point running average**

\[
y = \\
\text{Columns 1 through 7} \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.1429 \quad 0.2571 \\
\text{Columns 8 through 14} \\
0.3486 \quad 0.4217 \quad 0.4802 \quad 0.5270 \quad 0.5645 \quad 0.4516 \quad 0.3613 \\
\text{Columns 15 through 21} \\
0.2890 \quad 0.2312 \quad 0.1850 \quad 0.1480 \quad 0.1184 \quad 0.0947 \quad 0.0758
\]

In order to do a general derivation, we need the following formula:

\[
\sum_{k=N_1}^{N_2} r^k = \frac{r^{N_1} - r^{N_2+1}}{1 - r}, \quad \text{if } r \neq 1
\]

Or a simplified version:

\[
\sum_{k=0}^{N} r^k = \frac{1 - r^{N+1}}{1 - r}
\]
GENERAL DERIVATION

(1) For $n < 0$ \( y[n] = 0 \)
This is obvious because \( x[n] = 0, \ n < 0 \)

(2) When \( 0 \leq n < L \), the average takes in some points that are zero.

\[
y[n] = \frac{1}{L} \sum_{k=0}^{n} x[n-k] = \frac{a^n}{L} \sum_{k=0}^{n} (a^{-1})^k = \frac{a^n}{L} \frac{1 - (a^{-1})^{n+1}}{1 - a^{-1}}
\]

\[
= \frac{1}{L} \frac{a^n - a^{n+1}}{1 - a^{-1}} = \frac{1}{L} \frac{a^{n+1} - 1}{a - 1} = \frac{1}{L} \frac{1 - a^{n+1}}{1 - a}
\]

(3) When \( n \geq L \)

\[
y[n] = \frac{1}{L} \sum_{k=0}^{L-1} x[n-k] = \frac{1}{L} \sum_{k=0}^{L-1} a^{n-k}
\]

\[
= \frac{a^n}{L} \sum_{k=0}^{L-1} (a^{-1})^k = \frac{a^n}{L} \frac{1 - (a^{-1})^L}{1 - a^{-1}}
\]

\[
= a^n \left( \frac{1 - a^{-L}}{L - a^{-1}} \right) \text{this is a constant}
\]

.'.' in region #3, the output \( y[n] \)
will decay like \( a^n \).

in region #2, \( y[n] \) is rising.
RESPONSE to DECAYING $a^n$ INPUT

$L = 7$

RISING

DECAY LIKE $a^n$

TIME INDEX (n)