PROBLEM:
A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.

(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine the formula for the output signal $y[n]$.

Part A

\[ x[n] = 3e^{j\pi} e^{j0n} + 2e^{j\pi/2} e^{j0.3\pi n} + 2e^{-j\pi/2} e^{-j0.3\pi n} = -3 + 4\cos(0.3\pi n + \pi/2) \]

Part B

Nine-point averaging filter implies that


which means

\[ h[n] = \frac{1}{9}(\delta[n-4] + \delta[n-3] + \delta[n-2] + \delta[n-1] + \delta[n] + \delta[n+1] + \delta[n+2] + \delta[n+3] + \delta[n+4]). \]

The corresponding frequency response is given by

\[ H(\omega) = \frac{1}{9} \left( e^{-j\omega} + e^{-j3\omega} + e^{-j2\omega} + e^{-j\omega} + 1 + e^{j\omega} + e^{j2\omega} + e^{j3\omega} + e^{j4\omega} \right) \]

\[ = \frac{1}{9} \left( 1 + 2\cos(\omega) + 2\cos(2\omega) + 2\cos(3\omega) + 2\cos(4\omega) \right) \]

\[ H(0) = \frac{1}{9}(1 + 2 + 2 + 2 + 2) = 1 \]

\[ H(0.3\pi) = \frac{1}{9} \left( 1 + 2\cos(0.3\pi) + 2\cos(0.6\pi) + 2\cos(0.9\pi) + 2\cos(1.2\pi) \right) \]

\[ = \frac{1}{9} \left( 1 + 1.1755 - 0.6180 - 1.9021 - 1.6180 \right) = -0.2181 \]

\[ y[n] = -3(1) + 4(-0.2181)\cos(0.3\pi n + \pi/2) = -3 + 0.8724\cos(0.3\pi n - \pi/2) \]

If the nine-point averaging filter is constrained to be causal:


Then the frequency response contains an additional phase term:

\[ H(\omega) = \frac{1}{9} \left( 1 + 2\cos(\omega) + 2\cos(2\omega) + 2\cos(3\omega) + 2\cos(4\omega) \right) e^{-j4\omega} \]

and \( y[n] \) will be delayed by 4, because the filter’s impulse response is shifted right by 4.

\[ y[n] = -3 + 0.8724\cos(0.3\pi(n-4) - 0.5\pi) = -3 + 0.8724\cos(0.3\pi n + 0.3\pi) \]