PROBLEM:
The diagram in Fig. 1 depicts a cascade connection of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

![Figure 1: Cascade connection of two LTI systems.](image)

(a) Suppose that System #1 is a blurring filter described by the impulse response:

\[
    h_1[n] = \begin{cases} 
    0 & n < 0 \\ 
    \beta^n & n = 0, 1, 2, 3, 4, 5 \\ 
    0 & n > 5 
    \end{cases}
\]

and System #2 is described by the difference equation

\[
    y_2[n] = y_1[n] - \beta y_1[n - 1]
\]

Determine the impulse response function of the overall cascade system.

(b) Obtain a single difference equation that relates \(y[n]\) to \(x[n]\) in Fig. 1. Give numerical values of the filter coefficients for the specific case where \(\beta = \frac{1}{2}\).
CASCADE CONNECTION

\[
x[m] \xrightarrow{S_1} Y_1[m] \xrightarrow{S_2} Y[m]
\]

#1 \[ h_1[m] = \begin{cases} 0 & m < 0 \\ \beta^m & m = 0 \ 1 \ 2 \ 3 \ 4 \ 5 \\ 0 & m > 5 \end{cases} \]

#2 \[ h_2[m]: \]

\[
Y_2[m] = Y_1[m] - \beta Y_1[m-1] \\
h_2[m] = \delta[m] - \beta \delta[m-1]
\]

The cascade impulse response is determined by the convolution \( h_1 \ast h_2 \)

\[
\begin{array}{ccccccc}
 m & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
 h_1[m] & 1 & \beta & \beta^2 & \beta^3 & \beta^4 & \beta^5 & 0 \\
 h_2[m] & 1 & -\beta & 0 & 0 & 0 & 0 & 0 \\
 h_2[0]h_1[m] & 1 & \beta & \beta^2 & \beta^3 & \beta^4 & \beta^5 & 0 \\
 h_2[1]h_1[m-1] & 0 & -\beta & -\beta^2 & -\beta^3 & -\beta^4 & -\beta^5 & -\beta^6 \\
 h_2[2]h_1[m-2] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 y[m] & 1 & 0 & 0 & 0 & 0 & 0 & -\beta^6 \\
\end{array}
\]

Overall response: \( h[n] = \delta[n] - \beta^6 \delta[n-6] \)
(b) Difference Equation for system

\[ y[m] = x[m] - \beta^6 x[m-6] \]

\[ b_0 = 1 \]
\[ b_m = 0, \quad m \neq 0 \text{ or } 6 \]
\[ b_6 = -\beta^6 \]
\[ \beta = \frac{1}{2} \]
\[ b_6 = -\frac{1}{2^6} = -\frac{1}{64} \]

\[ y[m] = x[m] - \frac{1}{64} x[m-6] \]
Alternate Approach

\( y_1[n] = \sum_{k=0}^{M} b_k x[n-k] \)

\( b_k \) are the impulse response coefficients

\( b_k = 0 \quad k < 0 \)

\( \beta^k \quad k = 0, 1, 2, 3, 4, 5 \)

\( 0 \quad k > 5 \)

\( y_2[n] = y_1[n] - \beta y_1[n-1] \)

\( y_2[n] = \sum_{k=0}^{M} b_k x[n-k] - \beta \sum_{k=0}^{M} b_k x[n-1-k] \)

\( M = 5 \), \( b_k = \beta^k \)

\( y_2[n] = \sum_{k=0}^{5} \beta^k x[n-k] - \beta \sum_{k=0}^{5} \beta^k x[n-1-k] \)

\( y_2[n] = y[n] = x[n] - \beta^6 x[n-6] \)

\( \beta = \frac{1}{2} \)

\( y[n] = x[n] - \frac{1}{6} y x[n-6] \)

Note: Equation \( y[n] = x[n] - \beta^6 x[n-6] \)

Also follows from inspection of impulse response \( h[n] = s[n] - \beta^6 \delta[n-6] \)

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