PROBLEM:
The diagram in Fig. 1 depicts a cascade connection of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

\[
\begin{array}{cccccc}
& x[n] & \rightarrow & \text{LTI System #1} & h_1[n] & \rightarrow v[n] \\
& & & \text{LTI System #2} & h_2[n] & \rightarrow y[n]
\end{array}
\]

Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 has impulse response,

\[
h_1[n] = \begin{cases} 
0 & n < 0 \\
1 & n = 0 \\
-1 & n = 1 \\
0 & n > 1 
\end{cases}
\]

and System #2 is described by the difference equation

\[
y[n] = 0.25v[n] + 0.25v[n-1] + 0.25v[n-2] + 0.25v[n-3] \tag{1}
\]

(a) Determine the difference equation of System #1; i.e., the equation that relates \( v[n] \) to \( x[n] \).

(b) When the input signal \( x[n] \) is an impulse, \( \delta[n] \), determine the signal \( v[n] \) and make a plot. Show that the resulting output is the given impulse response \( h_1[n] \).

(c) From the difference equation in (1), determine \( h_2[n] \), the impulse response of System #2.

(d) Determine the impulse response of the overall cascade system, i.e., find \( y[n] \) when \( x[n] = \delta[n] \).

(e) From the impulse response of the overall cascade system as obtained in part (d), obtain a single difference equation that relates \( y[n] \) directly to \( x[n] \) in Fig. 1.
a) \( h[n] = s[n] - s[n-1] \)
\( v[n] = x[n] - x[n-1] \)

b) Let \( x[n] = s[n] \). Then \( v[n] = s[n] - s[n-1] \)

c) Let \( v[n] = s[n] \). Then \( y[n] = \frac{1}{4} s[n] + \frac{1}{4} s[n-1] + \frac{1}{4} s[n-2] + \frac{1}{4} s[n-3] \)

d) The impulse response associated with the cascade implies that \( x[n] = s[n] \), which results in \( v[n] = s[n] - s[n-1] \). Using this \( v[n] \) as input to system 2, we obtain
\[
y[n] = \frac{1}{4} s[n] + \frac{1}{4} s[n-1] + \frac{1}{4} s[n-2] + \frac{1}{4} s[n-3] - \frac{1}{4} s[n-4]
\]
\[
\therefore y[n] = \frac{1}{4} s[n] - \frac{1}{4} s[n-4]
\]

e) \( y[n] = \frac{1}{4} x[n] - \frac{1}{4} x[n-4] \)