PROBLEM:
Consider a system defined by

\[ y[n] = \sum_{k=0}^{M} b_k x[n - k] \]

(a) Suppose that the input \( x[n] \) is non-zero only for \( 0 \leq n \leq N - 1 \); i.e., it has a support of \( N \) samples. Show that \( y[n] \) is non-zero at most over a finite interval of the form \( 0 \leq n \leq P - 1 \). Determine \( P \) and the support of \( y[n] \) in terms of \( M \) and \( N \).

(b) Suppose that the input \( x[n] \) is non-zero only for \( N_1 \leq n \leq N_2 \). What is the support of \( x[n] \)? Show that \( y[n] \) is non-zero at most over a finite interval of the form \( N_3 \leq n \leq N_4 \). Determine \( N_3 \) and \( N_4 \) and the support of \( y[n] \) in terms of \( N_1, N_2, \) and \( M \).

Hint: Draw a sketch similar to Fig. 5.5 to illustrate the zero regions of the output signal.