Teaching Math Problem Solving Using a Web-based Tutoring System, Learning Games, and Students' Writing

Robert W. Maloy Sharon A. Edwards Gordon Anderson

University of Massachusetts Amherst

Abstract

This article describes the use of *4MALITY*, a web-based mathematics tutoring system, with 125 Massachusetts fourth graders and their teachers in three rural school districts during the 2007-2008 school year. At the outset, *4MALITY* was envisioned as a stand-alone web-based system providing tutorial support to individual students as they prepared for a statewide math achievement exam. What evolved was a broader set of strategies for teaching math problem solving using a combination of computer-based activities, learning games, and students' creative writing of math problems. These interventions proved successful, with 70%of students improving their performance from pre-test to post-test. Based on this study, we suggest that effective alternatives to whole class math instruction are available through structured practice and creative exploration of math problem solving using a creative blend of online and in-person learning activities.

Introduction

As she read the first question on a computer-based fourth grade math practice test, Molly looked up in bewilderment, unsure what to do next.

"What would the coaches tell you to do?" asked a college tutor who was working with the class that day.

Immediately relieved, Molly pulled paper and pencil next to the computer and began confidently writing and calculating through the rest of the questions.

The coaches mentioned in our opening story are four online math tutors named Estella Explainer, Chef Math Bear, How To Hound, and Visual Vera. They are found in 4MALI*TY* (*4th Grade Massachusetts Active Learning Intelligent Tutoring System*), a web-based tutoring system being developed by researchers from the University of Massachusetts Amherst for use by students in preparing for the Massachusetts fourth grade mathematics achievement test. It can be found at the 4mality log in page.

4MALITY's coaches were Molly's regular companions for 10 weeks during the 2007-2008 school year as 125 fourth graders in three rural communities used the system once a week in their public school classrooms. The project's overall goal was to improve the problem-solving and test-taking skills of students—both highly recommended math education reform goals (National Mathematics Advisory Panel, 2008; National Council of Mathematics, 2005). Our research focused on exploring whether a web-based tutoring system featuring problem-solving hints from friendly coaching characters might encourage students to spend more time working through math word problems strategically instead of simply clicking quickly to answer test questions. In addition, we wanted to explore how teachers might integrate an online tutoring system into their regular patterns of classroom math instruction.

Our study asked the following research questions:

(1) Do students show "growth in performance" from pretest to post-test after using the *4MALITY* system?

(2) Do students who show "growth in performance" from pre-test to post-test access more hints from online coaches than do students not showing "growth in performance?"

(3) What methods and strategies might teachers use to effectively integrate an online tutoring system into regular classroom practice and learning routines?

Initial results of the study showed cores improved from pre-test to post-test for 70% of the students. Moreover, by our observations, students began acting as thoughtful problem solvers. They regularly accessed the online coaches instead of quickly clicking answers. Additionally, students demonstrated in their writing of math problems that they could use math concepts (such as fractions and number operations) appropriately.

Background

The *4MALITY* Tutor

4MALITY is an online tutoring system, a type of technology that has been used to promote inquiry learning and problem solving among elementary and secondary school students (Karsenti, T., 2006; Arroyo, Walles, Beal, and Woolf, 2004a; Arroyo, I., Beal, C. R., Murray, T., Walles, R., and Woolf, B. P., 2004b; Merrill, D. C., Reiser, B. J., Ranney, M., and Trafton, J. G., 1997). The use of hints is an important feature of online tutors since it is possible to measure how often students access hints to support their learning (Feng, Heffernan, and Koedinger, 2006; Beal, C. R., Walles, R., Arroyo, I., and Woolf, B. P., 2007; Parshall, J., Kalohn, J., and Davey, T, 2001).

4MALITY is designed to teach mathematical problem-solving skills and test-taking strategies to fourth grade school children, with a particular emphasis on the Massachusetts Comprehensive Assessment System (MCAS) math exam. MCAS is a standardized test of math skills, presenting word problems that students must solve, in most instances, using multiple mathematical steps and number operations. Required of all fourth graders in the state's public schools, MCAS is part of how Massachusetts complies with the U. S. Government's No Child Left Behind (NCLB) Act.

4MALITY is implemented with the Java servlet architecture supported by a relational database instance. The user interface utilizes html and other scripting languages. The system contains MCAS-like test questions, annotated with hints and feedback designed to teach a wide range of problem solving techniques to students of various levels of knowledge. The grade level content spans from third through fifth grade.

A Problem Solving Model

4MALITY uses a hint model to organize suggestions and strategies along two axes: problem solving steps and learning style preferences. This model was created by a focus group of teaching educators and elementary school teachers.

There are five steps in the problem-solving axis, drawn from the work of George Polya (1973):

- Hint Level 1: What kind of question is this? Identifies the question type and connects it to known approaches.
- Hint Level 2: What is the question asking for? Highlights knowing what to look for in the problem and organizing to solve the problem.
- Hint Level 3: What do I already know that will help solve the problem? Focuses attention on pertinent information.
- Hint Level 4: What is my plan for solving the problem? Offers choice of a test-taking strategy and computation; breaking a problem down into smaller steps; using given information connected with what a student knows; making further deductions; and deciding if a chosen strategy will lead to a solution to the problem.
- Hint Level 5: How do I know I have solved the problem? Explores if a student has calculated what is being asked for by the question, eliminated obvious wrong answer choices, and checked the mathematical computation.

There are four categories of learning styles:

- (E)-explain the question in terms of the language used
- (M)-mathematical (domain-specific) computational operations
- (S)-test-taking and problem solving strategies
- (V)-visual approaches to computation

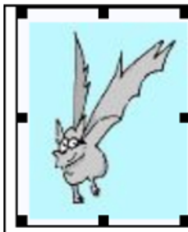	**Estella Explainer** is a language coach helping students to understand the vocabulary and structure of questions and terms.
	**Chef Math Bear** is a computational coach who solves problems using mathematical number operations.
	**How To Hound** is a strategy coach who suggests rounding, estimation, or eliminating obvious wrong answers as ways to solve problems.
	**Visual Vera** is a pictorial coach who illustrates problems with pictures, diagrams, charts, and other visual images.

Figure 1. Coaches in *4MALITY*

Virtual Coaches

For each problem in *4MALITY*, we have authored hints and suggestions from a particular problem solving "point of view." Four "virtual coaches"—intended to capture the "character" of each problem solving approach—represent these "points of view" (See Figure 1). Each coach has an iconographic representation that helps students identify its problem solving approaches. For example, Visual Vera offers a visual way to solve a question while How to Hound focuses on strategic solutions to problems. These coaches represent a distillation of our collective experience about how fourth graders approach solving the MCAS-style math problems.

Estella Explainer provides the hints at the first three levels, as in the following question from the 2004 MCAS 4th grade math test in the number sense learning standard:

- **"Gordon Stadium can seat 79,407 people while Hillcrest Stadium seats only 58,868 people. How many more people can Gordon Stadium seat than Hillcrest Stadium?"**

Figure 2. Estella Explainer Level 2 Hint for the Gordon Stadium Problem

The Gordon Stadium question can be confusing to students who think that "how many more" means they should add the numbers to solve the problem. Indeed, one of the answer choices is 138,275, the sum of 79,407 and 58,868. Estella explains that the question is asking HOW MANY MORE people can sit in the larger stadium (Figure 2).

At Level 4, all four coaches offer ideas for solving the problem; at Level 5 they present suggestions for reviewing and checking one's solution. Figure 3 shows Level 4 hints by Chef Math Bear, How To Hound, and Visual Vera for solving the Gordon Stadium problem. The highlighting emphasizes important information students need to know.

Two Types of Practice

The student interface design in *4MALITY* simulates the test-taking experience as closely as possible to offer maximal skill transfer. We provide two types of practice: "supported practice," where test questions are presented with hints and other annotations, and "test practice," where no support is offered with questions. This bi-level approach enables students to develop problem-solving skills in a supported way, and in so doing, build the confidence and positive mental attitude necessary to succeed on the timed tests. Many students fail to exhibit their full ability on standardized tests due to the environmental stresses of the test-taking experience. The test practice components provide an opportunity for students to apply and refine their skills under simulated test conditions (see Figure 4).

The system tracks student answers, records their responses, and gives teachers ongoing performance evidence of what a student knows and does not know.

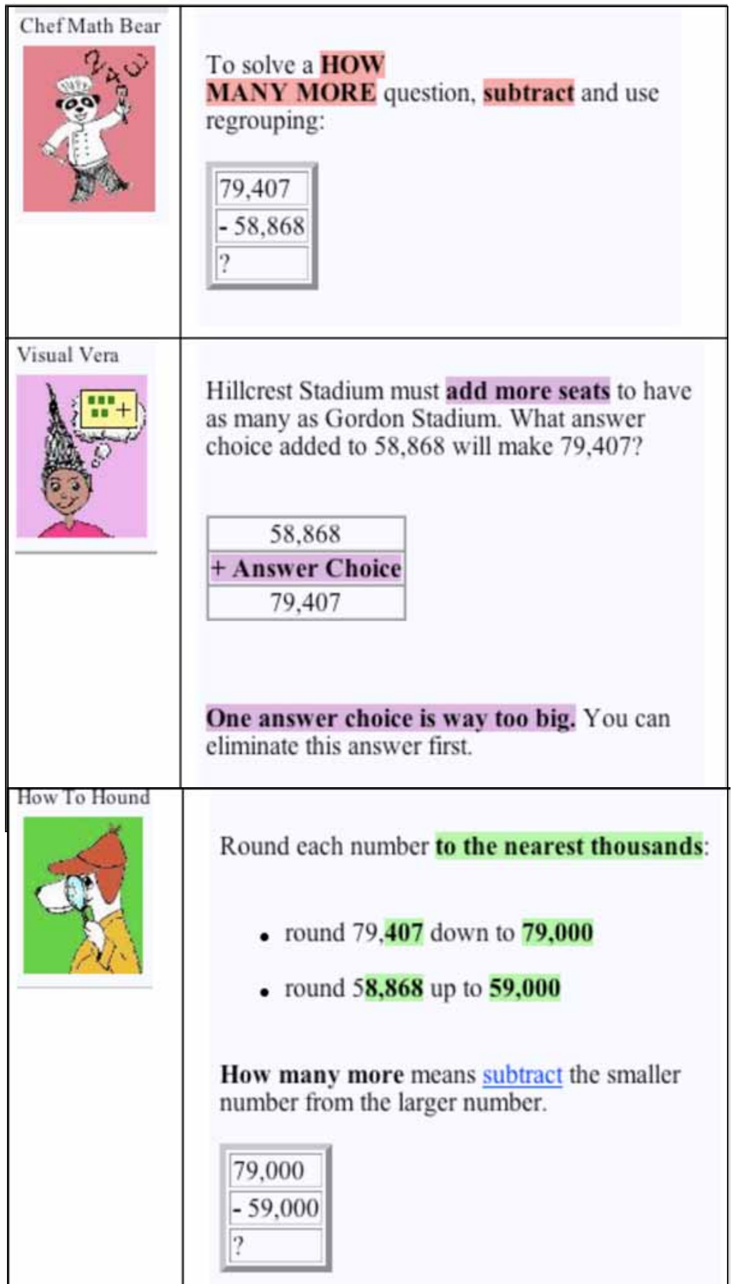

Figure 3. Hints from Coaches for the Gordon Stadium Problem

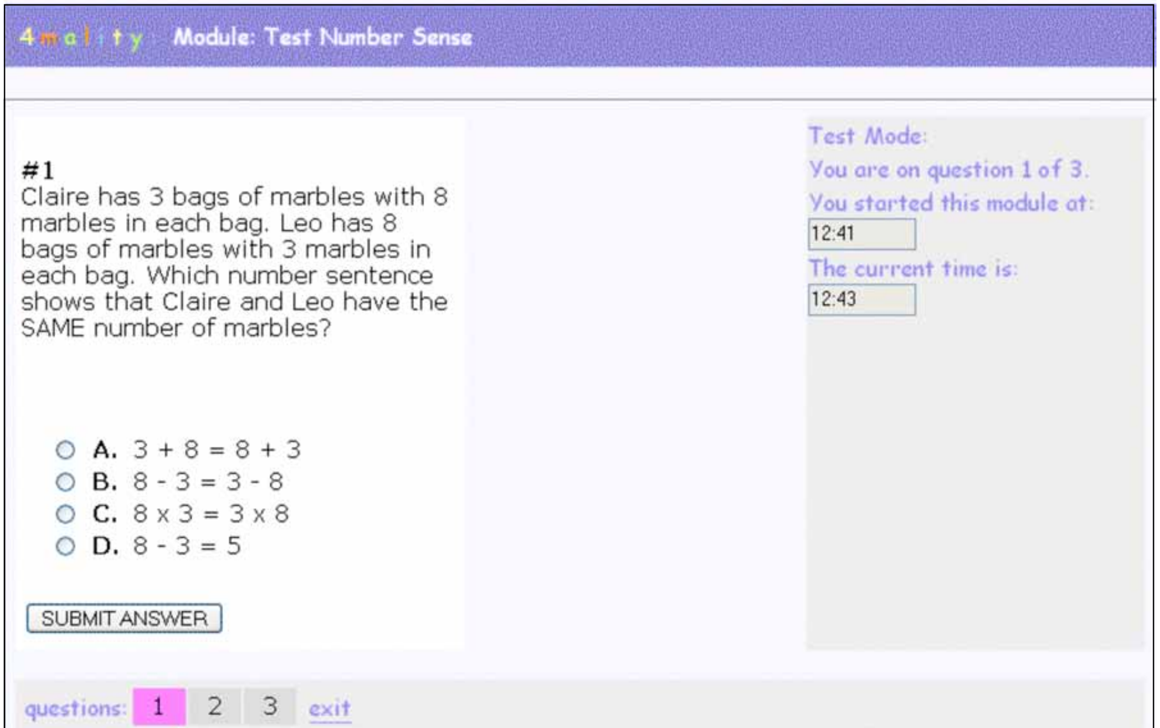

Figure 4: Test Page in *4MALITY*

Research Design

Sites

We used *4MALITY* in three rural school districts located in the Connecticut River Valley region of western Massachusetts. Fourth grade students in these school systems, like fourth grade students in Massachusetts as a whole, have struggled on MCAS (Massachusetts Comprehensive Assessment System). Statewide, 52% of the state's 71,000 fourth grade students scored at the "warning" (13 percent) or "needs improvement" (39 percent) performance level on 2006-2007 math MCAS test (Massachusetts Department of Education, 2008). The rates of students scoring in the "warning" and "needs improvement" categories of the three districts in the study were 48%, 56% and 67% respectively.

Sample

Five fourth grade classrooms totaling 125 students participated in the project. Teachers agreed to use *4MALITY* for a minimum of ten weeks during either math or computer instruction time. At the completion of the initial study, two of the classrooms continued using *4MALITY* for an additional four weeks as a self-selected choice for independent learning. In two schools, *4MALITY* was first introduced to small groups of students in a multi-grade after-school math club, and, after a two month trial, made available to all students during regular school time. A computer lab in each school, with Internet-connected computers, was used for the project.

Procedures

With slight variations from school to school, we followed a basic pattern for using the system with students.

(1) After explaining the project to teachers and administrators, we introduced the system to the students as a whole class. Accompanied by 3-D stuffed animals to represent the online coaches, we showed a sample math MCAS question and asked the students how they might solve the problem. Invariably, the children suggested different ideas and approaches, usually corresponding to the language, computational, strategic, or visual hints offered by the online coaches.

(2) The online coaches, we told the students, are here to help you with the questions by offering ideas and suggestions while not giving the answers. Being successful with math problem solving, we explained, is the same as being successful in sports, dance, music, or video games—you need focus, energy, practice, and support. Good math thinkers learn from good math coaches

who provide hints that resolve confusions and activate prior knowledge. Like a theater coach who stands off stage and offers lines to actors during rehearsals, math coaches provide support as needed to students practicing math skills.

(3) We explained that high stakes math tests, unlike day-to-day math activities in school, ask students to apply their math knowledge in specific ways under special conditions. On a test, there are no outside coaches. Ideas and suggestions must be inside the student's head, ready to use when the question demands it. Tests are not a race to see who identifies the answer quickest; they are identifying who understands what is being asked in a problem, develops a plan for solving the problem, follows the plan, and arrives at a correct solution. *4MALITY*, we assured the students, is a way for you to prepare for high stakes tests and practice math problem solving with the help of there-when-you-need-them coaches.

(4) Following our introduction, students were given a ten-question pre-test coaching suggestions that used MCAS questions from previous years.

(5) After completing the pretest, students worked individually to complete six practice modules, each consisting of four questions. In the practice modules, students could view as many coaching suggestions as they wanted to before answering the question.

(6) An onscreen scoreboard kept track of points earned for each correct answer. To increase motivation and to help the students to focus on the questions, the scoreboard awarded:

- 8 points for a correct answer on the first try.
- Fewer points for a corrected answer—the number of points depended on whether or not coaches were consulted before choosing a new answer.

To reinforce the importance of checking with coaches before submitting answers, and to counter a desire to "game" the system to determine the right answer, the system differentiates points awarded for a corrected answer choice (that is, an answer chosen after submitting a wrong answer). If a student consults one or more coaches before submitting a correct second answer, more points are awarded than if no coach is consulted. When a student finishes each problem, the system issues a congratulatory statement and a reminder to continue consulting the coaches.

(7) When students finished the six modules, they took a post-test without coaching hints.

(8) In two of the three schools, students did not work exclusively with *4MALITY* during math or computer instructional time. A series of small group rotations occurred where some students worked on the *4MALITY* system and other students engaged in related math problem solving activities using computers, games, or creative writing. The students then exchanged groups so that everyone had time on *4MALITY* and time with the other activities. These related math activities included:

- **Computer Math Games.** We chose two interactive educational websites—*Rainforest Maths* (www.rainforestmaths.com) by Australian educator Jennie Eather and the *National Library of Virtual Manipulatives* from Utah State University (http://nlvm.usu.edu/)—to compliment the *4MALITY* system when students were in the computer lab. These sites offer engaging online math practice in game-like formats. Each game allows students to select the level of difficulty they want to try. A math-confident student can play above the 4th grade level while a less math proficient individual can practice at or below grade level, if needed. Each site offers games for specific math concepts such as probability, estimation, fractions, number operations, and so on. These computer-based games also proved invaluable when some students finished their *4MALITY* modules sooner than others and needed other activities before returning to their classroom. *Rainforest Maths* is presently off-line due to unauthorized referencing and downloading.

- **Math Board Games.** Math board games offered another way to engage students in problem solving. *Qwirkle*, a game from MindWare, was especially popular. In *Qwirkle*, individuals or teams must arrange small, square blocks with different shaped and color designs on a flat surface in rows of up to six. We had small groups play *Qwirkle* while other members of the class used *4MALITY*.

- **Creative Writing by Students**. Inviting students to write their own math problems was another way we taught math problem solving. As a recent study by the Pew

Internet & American Life Project and the National Commission on Writing noted, children enjoy opportunities to write creatively, particularly when they can choose topics that are of interest to them (Lenhart, Arafeh, Smith & Macgill, 2008). We wanted to assess how well students understood math concepts by the way they used them in their word problems and math comics. The students embraced the idea of math writing, authoring problems that used child-engaging language and incorporated child-familiar topics such as pets, food, music, sports, and shopping. One example is Kelsea's fraction problem: "There were 50 Labrador puppies. 16 were chocolate, 12 were yellow, and 22 were black. Then I got 16 golden retriever puppies. There were 66 puppies. What fraction of the total number of puppies were chocolate, yellow and golden?" As teachers introduced concepts of fractions, probability, estimation, and measurement, students eagerly wrote questions for their classmates to answer.

Findings

Descriptive Statistics

Table 1 summarizes the classes used in the statistical analysis of the study. Only those students who completed the pre- and post-tests and participated in the program activities are included.

Significance Test for Pre and Post Tests

The pre and post test items were chosen from past MCAS tests. The items were chosen so that the difficulty of the items on the pre test was matched by those chosen for the post test according to item statistics for the MCAS tests. We acknowledge that this method does not ensure that the actual level of difficulty experienced by students in our study were equal from pre to post test. We make the assumption that students in our study would experience the pre and post tests as having approximately equal difficulty. The descriptive statistics if the pre and post tests are presented in table 2.

Pre Test Scores	Post Test Scores
Min. 1	Min. 1
1st Qu. 3	1st Qu. 6
Median 4	Median 7
Mean 4.384	Mean 6.68
3rd Qu. 6	3rd Qu. 8
Max. 9	Max. 9
SD 1.958	SD 1.807

Table 2. Pre and Post Test Statistics N=125

We ran a two-tailed matched-pair t-test to determine if the pre test scores were statistically different from the post test scores for the students in the study.

We calculated a t value of -12.58, which is highly significant ($p < .01$). We therefore claim that the pre and post test scores show a statistically significant gain.

Although we claim a statistically significant gain in test scores, we cannot claim that this was due solely to the effect of our program since we do not have a control group.

Average Growth Per Class

The average growth per class group is shown in Table 3.

Class Number	Number of Students*
Class 1	20
Class 2	37
Class 3	21
Class 4	23
Class 5	24
Total	**125**

* completed pre-test, math activities, and post-test.

Table 1. Classes in Study

Class	Average Growth
1	26.67
2	27.63
3	20.11
4	24.64
5	26.85

Table 3. Growth by Class

An analysis of variance showed that these averages are not significantly different.

Connection between Hints and Performance

Since several types of math-related activities were used in the intervention, we investigated what effect the use of *4MALITY* had in determining outcomes. We took the number of hints accessed by students as the measure of the quantity of *4MALITY* treatment received. The effect of *4MALITY* treatment can then be assessed by the correlation between the number of hints received and the growth in performance. A positive correlation would imply that *4MALITY* had a positive effect on performance. A regression model was constructed using the pretest and number of hints accessed as independent variables and the posttest as the dependent variable. While the pretest did significantly contribute to the prediction of the posttest score ($p<.01$), the number of hints accessed did not ($p<.45$). We speculate that students used the hints more or less uniformly, and therefore the number of hint accesses does not help differentiate the outcome scores. We are currently investigating the system logs to see if a pattern of usage that correlates with outcomes can be determined.

Conclusions

In this study, we sought to assess the impact of the *4MALITY* web-based tutoring system on problem solving and test taking skills of 125 fourth graders in three rural communities. We had the reality of yearly statewide, high-stakes standards-based achievement tests as an immediate context for our study. We asked the following research questions:

(1) Would students show a growth in performance from pre-test to post-test after using *4MALITY*?

(2) What suggestions could we give to teachers about integrating computer-based learning resources like *4MALITY* into their regular patterns of math instruction?

The first we examined statistically and the second we explored qualitatively through classroom observations. Our findings can be summarized as follows:

Growth in Performance

We found a mean gain of 25.51% in test scores from pre-test to post-test among all student participants, while 36 student participants registered gains of 40% or more from pre-test to post-test. The difference between pretest and post-test score distributions was found to be statistically highly significant ($p<.01$).

Integrating Technology into Classroom Math Instruction

In two of the schools (three of the five classrooms in the study), we found that student work on the *4MALITY* computer system could be successfully combined with individual and small group activities involving computers, learning games and creative writing. In these classrooms, students not only used the online tutoring system, but also accessed learning games on interactive educational websites, played math-themed board games, and wrote their own math problems and math comics. At these times, there was a minimum of whole group math instruction. Different groups of students engaged in different activities at the same time, rotating between online and in-person activities in small learning groups.

Combining technology and non-technology activities in this way was a new approach for the teachers who had told us when we began the study, they imagined all the students would be using computers at the same time. They had not thought of how computers in general, and *4MALITY* in particular, could serve as a hub of series of interactive math learning experiences.

The result was a model for diversifying teachers' regular approach to math instruction using technology. Web-based and in-person learning experiences functioned like stretchable elastic bands that could be expanded or contracted to fit the needs of the classroom. Teachers might choose to do more of one activity and less of another as they deem necessary, but a combination provides students with useful experiences that add to their knowledge of math and their readiness to succeed on high stakes tests. One teacher suggested to us that she thought interactive educational websites, board games, and students' creative writing could all be used to support math learning, even without the use of an online math tutor like *4MALITY*.

The combination of online and in-person learning activities used in two of the schools also showed evidence of supporting the development of a problem solving mindset among students. Students began proceeding thoughtfully and strategically when solving math problems in class or on tests; what we call moving from "clicking to checking." We observed students consulting *4MALITY*'s coaches before answering, carefully considering answer choices before responding to questions, and spending time to write their own math problems to present to classmates. We intend to investigate whether a longer period of sustained practice using a Web-based tutor, learning games, and creative writing could make a problem-solving mindset a more permanent feature of how young students approach math learning.

References

Anno, M. (1995). *Anno's magic seeds*. New York: Pihilomel.

Anno, M. (1999). *Anno's mysterious multiplying jar*. New York: Putnam Juvenile.

Arroyo, I., Woolf, B., and Beal, C.R. (2006). Addressing cognitive differences and gender during problem solving. *International Journal of Technology, Instruction, Cognition and Learning*, 4, 31-63.

Arroyo, I., Walles, R., Beal, C. R., and Woolf, B. P. (2004a). *Effects of Web-based tutoring software on students' math achievement*. Paper presented at the meeting of the American Educational Research Association, San Diego, CA.

Arroyo, I., Beal, C. R., Murray, T., Walles, R., and Woolf, B. P. (2004b). Web-based intelligent multimedia tutoring for high stakes achievement tests. In J. C. Lester, R. M. Vicari, & F. Paraguaçu (Eds.), *Intelligent Tutoring Systems, 7th International Conference, ITS 2004* (pp. 468-477). Maceiò, Alagoas, Brazil.

Beal, C. R., Walles, R., Arroyo, I., and Woolf, B. P. (2007). Online tutoring for math achievement testing: A controlled evaluation. *Journal of Interactive Online Learning, 6*(1), 43-55.

Feng, M., Heffernan, N. T., and Koedinger, K. R. (2006). Predicting state test scores better with intelligent tutoring systems: Developing metrics to measure assistance required. In Ikeda, Ashley and Chan (Eds.). *Proceedings of the 8th International Conference on Intelligent Tutoring Systems.* Berlin: Springer-Verlag, 31-40.

Karsenti, T. (2006). Effects of different tutoring modalities in online learning environments. In T. Reeves & S. Yamashita (Eds.), *Proceedings of World Conference on E-Learning in Corporate, Government, Healthcare, and Higher Education 2006* (pp. 1816-1821). Chesapeake, VA: AACE.

Lenhart, A., Arafeh, S., Smith, A., and Macgill, A. R. (2008, April). *Writing, technology and teens*. Washington, DC: Pew Internet & American Life Project.

Massachusetts Department of Education. (2008). *2007 MCAS report (district) for grade 04 all students*. Malden, MA: Author. Retrieved June 9, 2008, from http://profiles1.doe.mass.edu/mcas.aspx

Merrill, D. C., Reiser, B. J., Ranney, M., and Trafton, J. G. (1997). Effective tutoring techniques: A comparison of human tutors and intelligent tutoring systems. *Journal of the Learning Sciences, 2*(3), 277-305.

National Council of Teachers of Mathematics. (2005, May). *Computation, calculators, and common sense*. Reston, VA: National Council of Teachers of Mathematics.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U. S. Department of Education.

Parshall, J., Kalohn, J., and Davey, T. (2001). *Practical considerations in computer-based testing*. New York: Springer.

Polya, G. (1973). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.

Tang, G. (2005). *Math for all seasons*. New York: Scholastic Paperbacks

Tang, G. (2004a). *Grapes of math*. New York:

Scholastic Paperbacks.
Tang, G. (2004b). *Math fables*: New York: Scholastic Press

Tang, G. (2002). *The best of times*. New York: Scholastic Press

Robert W. Maloy is a Senior Lecturer in the School of Education at the University of Massachusetts Amherst. His latest book is Transforming Learning with New Technologies to be published by Allyn & Bacon in 2011.

Sharon A. Edwards is a clinical faculty member in the School of Education at the University of Massachusetts Amherst. She has co-authored three books with Robert W. Maloy including Transforming Learning with New Technologies.

Gordon Anderson is a software designer and doctoral student in the Computer Science Department at the University of Massachusetts Amherst. He has been the principal software engineer for the 4MALITY tutoring system.