The Cantor Set

NAME___________________________

When working with sets, the following notations are used.

Interval Notation: $[a,\ b) = \{x \mid a \le x < b\}$

Compliment of a set S, S': $S' = \{x \mid x \notin S\}$

Intersection of a sequence of sets: $\bigcap_{k=1}^{3} S_k = S_1 \cap S_2 \cap S_3$

Constructing the Cantor Set

Begin with set $C_1 = [0,1]$.

1. What is the length of C_1?

Now remove the open middle third of this interval, (1/3, 2/3), leaving two closed intervals behind.

2. This will be set C_2.

(a) What is the length of C_2, the union of both subintervals?

(b) Write C_2 in interval notation.

(c) $C_1 \cap C_2 =$ ________________________

(d) $C_2' =$ ____________________________

3. Repeat the procedure, removing the open middle third of each of the sub-intervals in C_2 to get four closed intervals, C_3.

(a) What is the length of C_3?

(b) Write C_3 in interval notation.

(c) $C_1 \cap C_2 \cap C_3 =$ ____________________ $C_3' =$ ____________________________________

This process can be repeated indefinitely by removing the open middle third of each of the sub-intervals to get a new set.

4. Consider C_4.

(a) Write C_4 in interval notation.

(b) How many closed intervals are in the set C_4?

(c) What is the length of C_4?

5. Consider C_5**.**

(a) How many closed intervals are in the set C_5?

(b) What is the length of C_5?

6. Consider C_6.

(a) How many closed intervals are in the set C_6?

(b) What is the length of C_6?

7. $\bigcap_{k=1}^{6} C_k =$ ____________________

8. This construction can be extended for any positive integer, *k*.

(a) How many intervals are in the set C_k?

(b) What is the length of C_k?

(c) Write the first three terms of the interval notation representation of C_k.

(d) $\bigcap_{k=1}^{n} C_k =$ __________________

Properties of the Cantor Set

9. Defining the sets, C_k, of intervals in this way creates a sequence of sets. Taking the infinite intersection $\bigcap_{k=1}^{\infty} C_k$ of all elements in the sequence defines a new set called the **Cantor Set**.

(a) Verify that the Cantor Set is not an empty set.

(b) Find three different values contained in the Cantor set.

10. How many elements are in the Cantor Set?

11. What is the length of the Cantor Set?