PROBLEM:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = 3x[n] - 2x[n-1] + 2x[n-2] - 3x[n-4].$$

(a) Draw a block diagram that represents this system in terms of unit-delay elements, coefficient multipliers, and adders.

(b) Determine the impulse response $h[n]$ for this system. Express your answer as a sum of scaled and shifted unit impulse sequences.

(c) Use convolution to determine the output due to the input

$$x[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$$

Plot the output sequence $y[n]$ for $-3 \le n \le 10$.

(d) Now consider another LTI system whose impulse response is

$$h_d[n] = \delta[n] + 2\delta[n-1] + \delta[n-2].$$

Use convolution again to determine $y_d[n] = x_d[n] * h_d[n]$, the output of this system when the input is

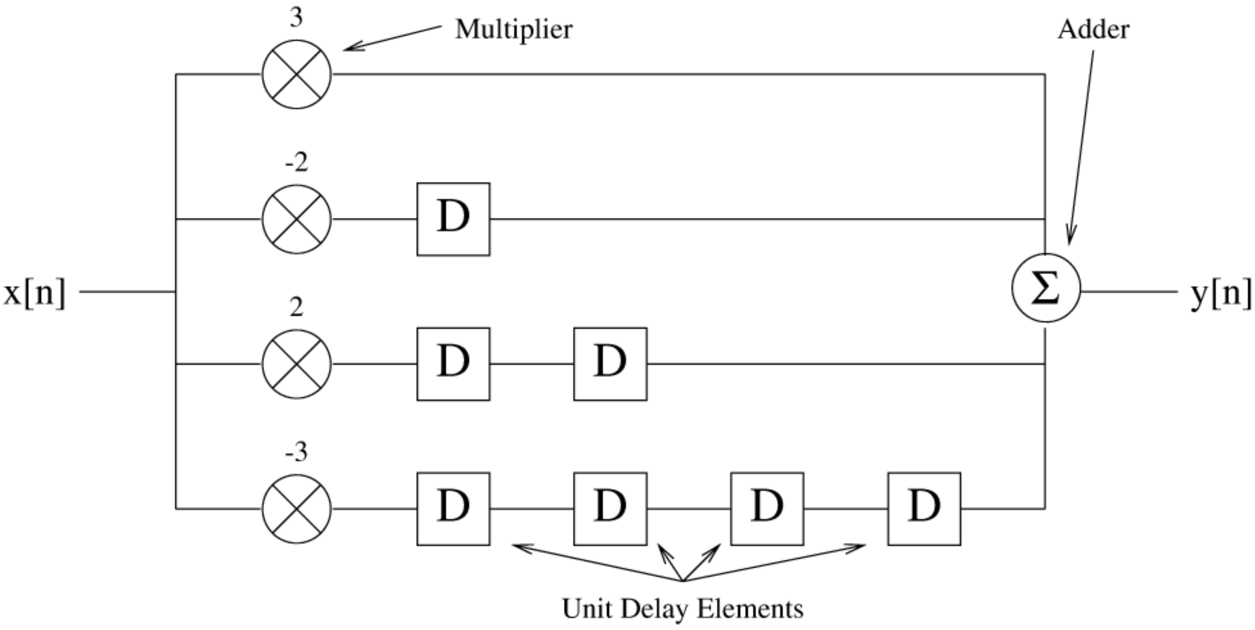
$$x_d[n] = 3\delta[n] - 2\delta[n-1] + 2\delta[n-2] - 3\delta[n-4].$$

How does your answer compare to the answer in part (c)? This example illustrates the general commutative property of convolution; i.e., $x[n] * h[n] = h[n] * x[n]$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

Part A



Part B

Plugging $x[n] = \delta[n]$ into the difference equations yields the output

$$h[n] = 3\delta[n] - 2\delta[n-1] + 2\delta[n-2] - 3\delta[n-4]$$

Part C

$$\begin{aligned}
y[n] &= x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \\
&= \sum_{k=0}^{2} x[k]\, h[n-k] \qquad \text{(since } x[k] = 0 \text{ for } k < 0 \text{ and } k > 2\text{)} \\
&= x[0]\, h[n] + x[1]\, h[n-1] + x[2]\, h[n-2] = h[n] + 2\, h[n-1] + h[n-2] \\
&= 3\,\delta[n] - 2\,\delta[n-1] + 2\,\delta[n-2] - 3\,\delta[n-4] + \\
&\quad\; 6\,\delta[n-1] - 4\,\delta[n-2] + 4\,\delta[n-3] - 6\,\delta[n-5] + \\
&\quad\; 3\,\delta[n-2] - 2\,\delta[n-3] + 2\,\delta[n-4] - 3\,\delta[n-6] \\
&= \boxed{3\,\delta[n] + 4\,\delta[n-1] + \delta[n-2] + 2\,\delta[n-3] - \delta[n-4] - 6\,\delta[n-5] - 3\,\delta[n-6]}
\end{aligned}$$

McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

Part D

$$
\begin{aligned}
y_d[n] &= x_d[n] * h_d[n] = \sum_{k=-\infty}^{\infty} x_d[k]\, h_d[n-k] \\
&= \sum_{k=0}^{4} x_d[k]\, h_d[n-k] \qquad \text{(since } x_d[k] = 0 \text{ for } k < 0 \text{ and } k > 4) \\
&= x_d[0]\, h_d[n] + x_d[1]\, h_d[n-1] + x_d[2]\, h_d[n-2] + x_d[3]\, h_d[n-3] + x_d[4]\, h_d[n-4] \\
&= 3\, h_d[n] - 2\, h_d[n-1] + 2\, h_d[n-2] - 3\, h_d[n-4] \\
&= 3(\delta[n] + 2\,\delta[n-1] + \delta[n-2]) - 2(\delta[n-1] + 2\,\delta[n-2] + \delta[n-3]) + \\
&\quad\; 2(\delta[n-2] + 2\,\delta[n-3] + \delta[n-4]) - 3(\delta[n-4] + 2\,\delta[n-5] + \delta[n-6]) \\
&= \boxed{3\,\delta[n] + 4\,\delta[n-1] + \delta[n-2] + 2\,\delta[n-3] - \delta[n-4] - 6\,\delta[n-5] - 3\,\delta[n-6]} \\
&= \text{Same answer as part (c).}
\end{aligned}
$$

McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.