PROBLEM:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

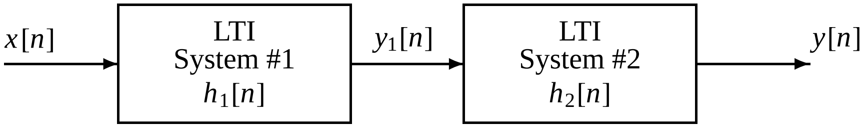

Figure 1: Cascade connection of two LTI systems.

(a) Suppose that System #1 is a "blurring" filter described by the difference equation

$$y_1[n] = \sum_{k=0}^{6} \beta^k x[n-k],$$

and System #2 is described by the impulse response

$$h_2[n] = \delta[n] - \beta\delta[n-1],$$

where β is a real number. Determine the impulse response sequence, $h[n] = h_1[n] * h_2[n]$, of the overall cascade system.

(b) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1. Give numerical values of the filter coefficients for the specific case where $\beta = \frac{1}{2}$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

We need to compute $h_1[n] * h_2[n]$, and we start with $h_2[n] = \delta[n] - \beta\delta[n-1]$. Therefore,

$$
\begin{aligned}
h_1[n] * h_2[n] &= h_1[n] * (\delta[n] - (\beta)\delta[n-1]) \\
&= h_1[n] * \delta[n] - (\beta)h_1[n] * \delta[n-1] \;=\; h_1[n] - \beta h_1[n-1]
\end{aligned}
$$

We get the following partial expressions as

$$
\begin{aligned}
h_1[n] &= \sum_{k=0}^{6} \beta^k \delta[n-k] \\
\beta h_1[n-1] &= \sum_{k=0}^{6} \beta^{k+1} \delta[n-k-1]
\end{aligned}
$$

which results in

$$
\begin{aligned}
h_1[n] * h_2[n] &= \sum_{k=0}^{6} \beta^k \delta[n-k] - \sum_{k=0}^{6} \beta^{k+1} \delta[n-k-1] \\
&= \sum_{k=0}^{6} \beta^k \delta[n-k] - \sum_{k=1}^{7} \beta^k \delta[n-k] \\
&= \delta[n] + \left(\sum_{k=1}^{6} \beta^k \delta[n-k] - \sum_{k=1}^{6} \beta^k \delta[n-k] \right) - \beta^7 \delta[n-7] \\
&= \delta[n] - \beta^7 \delta[n-7]
\end{aligned}
$$

(b) Now we can use convolution to write a general expression that relates $y[n]$ to $x[n]$:

$$
\begin{aligned}
y[n] &= (h_1[n] * h_2[n]) * x[n] \\
&= (\delta[n] - \beta^7 \delta[n-7]) * x[n] \\
&= \delta[n] * x[n] - \beta^7 \delta[n-7] * x[n] \\
&= x[n] - \beta^7 x[n-7]
\end{aligned}
$$

If $\beta = 0.5$, then $\boxed{y[n] = x[n] - 0.0078125x[n-7].}$

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