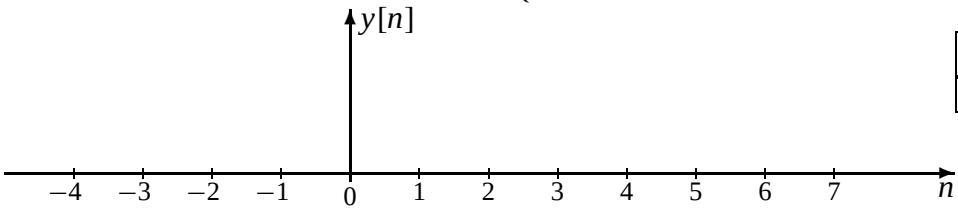
PROBLEM:

(a) If the filter coefficients of an FIR filter are $\{b_k\} = \{0, 1, -2, 1\}$, make a plot of the output when the input is the unit step signal: $x[n] = u[n] = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$



Label Carefully

Plot zero values also

(b) Suppose that the frequency response of a different FIR filter is

$$\mathcal{H}(\hat{\omega}) = \cos(\tfrac{1}{2}\hat{\omega})e^{-j\hat{\omega}}$$

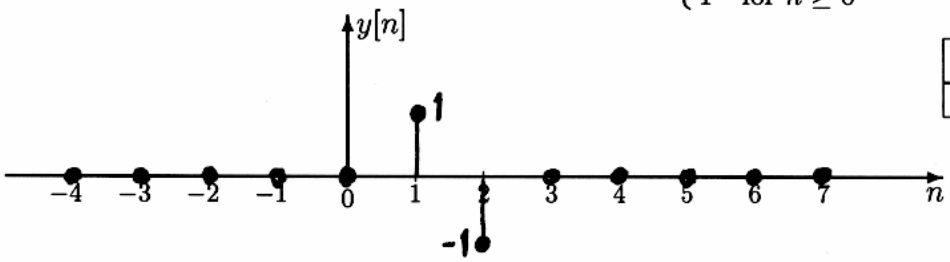
If the input signal is $x[n] = 7 + 2\cos(0.5\pi n + \pi)$ for $-\infty < n < \infty$, determine a simple mathematical expression for the output signal $y[n]$.

$y[n] =$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) If the filter coefficients of an FIR filter are $\{b_k\} = \{0, 1, -2, 1\}$, make a **plot** of the output when the input is the unit step signal: $x[n] = u[n] = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$



Label Carefully
Plot zero values also

Convolution:

1 1 1 1 1 1 1 . . .
0 1 -2 1 0 0 . . . .

0 1 1 1 1 1 1 1 . . . .
-2 -2 -2 -2 -2 -2 . . . .
1 1 1 1 1 . . . .

0 1 -1 0 0 0 0 . . . .

n=0 n=1

$y[n] = \delta[n-1] - \delta[n-2]$

(b) Suppose that the frequency response of a different FIR filter is

$$\mathcal{H}(\hat{\omega}) = \cos(\tfrac{1}{2}\hat{\omega})e^{-j\hat{\omega}}$$

If the input signal is $x[n] = 7 + 2\cos(0.5\pi n + \pi)$ for $-\infty < n < \infty$, determine a simple mathematical expression for the output signal $y[n]$.

$y[n] = 7 + \sqrt{2}\cos(0.5\pi n + \pi/2)$

x[n] has two frequency components: $\hat{\omega} = 0$ & $\hat{\omega} = 0.5\pi$

$\mathcal{H}(\hat{\omega})\big|_{\hat{\omega}=0} = \mathcal{H}(0) = \cos(0)e^{-j0} = 1$

$\mathcal{H}(0.5\pi) = \cos(0.25\pi)e^{-j0.5\pi} = \frac{1}{2}\sqrt{2}\, e^{-j\pi/2}$

$y[n] = 7\times\mathcal{H}(0) + 2\times|\mathcal{H}(0.5\pi)|\cos(0.5\pi n + \pi + \angle\mathcal{H}(0.5\pi))$

$= 7(1) + 2(\frac{1}{2}\sqrt{2})\cos(0.5\pi n + \pi - \pi/2)$