PROBLEM:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

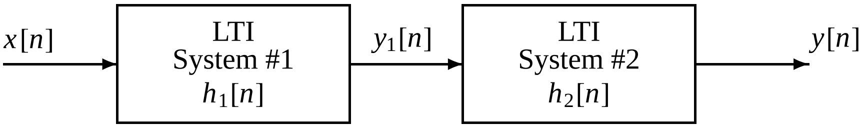


Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is an FIR filter described by the impulse response:

$$h_1[n] = \begin{cases} 0 & n < 0 \\ 2^n & n = 0, 1, 2, 3, 4, 5 \\ 0 & n > 5 \end{cases}$$

and System #2 is described by the difference equation

$$y_2[n] = y_1[n] - 2y_1[n-1]$$

(a) Determine the filter coefficients of System #1, and also for System #2.

(b) When the input signal $x[n]$ is an impulse, $\delta[n]$, determine the signal $y_1[n]$ and make a plot.

(c) Determine the impulse response of System #2.

(d) Determine the impulse response of the overall cascade system, i.e., find $y[n]$ when $x[n] = \delta[n]$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

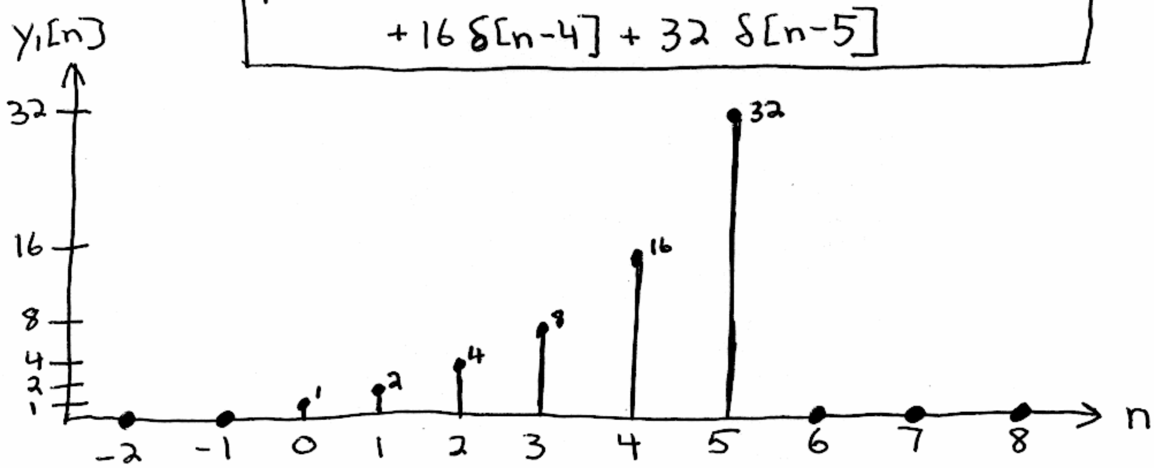
a) Filter coefficients, $a_0 = 2^0,\ a_1 = 2^1,\ a_2 = 2^2,\ a_3 = 2^3,\ a_4 = 2^4,\ a_5 = 2^5$

For system #1 : $\boxed{a_0 = 1,\ a_1 = 2,\ a_2 = 4,\ a_3 = 8,\ a_4 = 16,\ a_5 = 32}$

For system #2 : $\boxed{b_0 = 1,\ \ b_1 = -2}$

b) When $x[n] = \delta[n]$, then $y_1[n] = h_1[n]$.

Thus, $$\boxed{\begin{aligned} y_1[n] = {}& \delta[n] + 2\delta[n-1] + 4\delta[n-2] + 8\delta[n-3] \\ & + 16\delta[n-4] + 32\delta[n-5] \end{aligned}}$$



c) $h_2[n] = b_0\,\delta[n] + b_1\,\delta[n-1] = \boxed{\delta[n] - 2\delta[n-1]}$

(simply plug in $y_1[n] = \delta[n]$)

d) $h[n]$ = impluse response of cascade system $= h_1[n] * h_2[n]$

$$= h_2[n] * h_1[n] = \sum_{k=-\infty}^{\infty} h_2[k]\, h_1[n-k]$$

$$= h_2[0]\, h_1[n] + h_2[1]\, h_1[n-1]$$

$$= h_1[n] - 2 h_1[n-1]$$

$$\begin{aligned} = {}& (\delta[n] + 2\delta[n-1] + 4\delta[n-2] + 8\delta[n-3] + 16\delta[n-4] + 32\delta[n-5]) \\ & -2(\delta[n-1] + 2\delta[n-2] + 4\delta[n-3] + 8\delta[n-4] + 16\delta[n-5] + 32\delta[n-6]) \end{aligned}$$

$$= \boxed{\delta[n] - 64\,\delta[n-6]}$$