PROBLEM:

This problem is concerned with finding the output of an FIR filter for a given input signal. A linear time-invariant system is described by the difference equation

$$y[n] = \sum_{k=0}^{5} k x[n-k]$$

The input to this system is *unit step* signal, denoted by $u[n]$, i.e., $\quad x[n] = u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$

(a) Determine the filter coefficients $\{b_k\}$ of this FIR filter.

(b) Find the impulse response, $h[n]$, for this FIR filter. The impulse response is a discrete-time signal, so make a (stem) plot of $h[n]$ versus n.

(c) Find the output $y[n]$ when the input is $u[n]$, and make a plot of $y[n]$ vs. n. (Hint: you might find it useful to check your results with MATLAB's conv( ) function.)
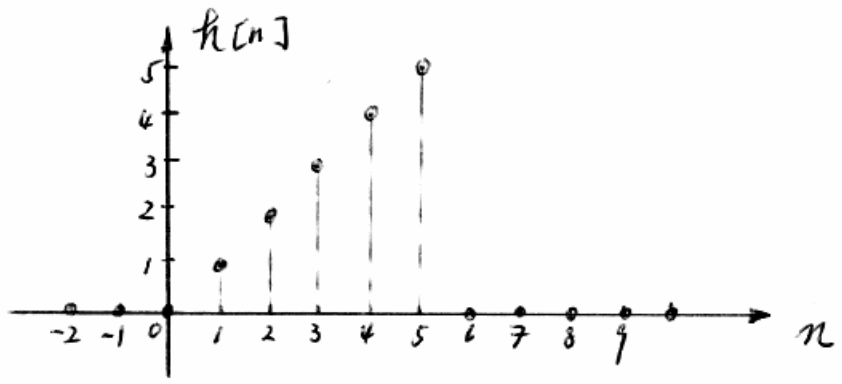
SOLUTION

(a)

$$b_k = \begin{cases} k & k = 0, 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

(b)

$$h[n] = b_n = \begin{cases} n & n = 0, 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

(c)

$$y[n] = \sum_{k=0}^{5} k\, x[n-k] = \sum_{k=0}^{\max(5,n)} k \implies$$

n	≤ 0	0	1	2	3	4	5	>5
$y[n]$	0	0	1	3	6	10	15	15