PROBLEM:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

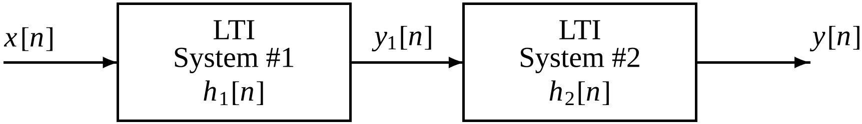

Figure 1: Cascade connection of two LTI systems.

(a) Suppose that System #1 is a blurring filter described by the impulse response:

$$h_1[n] = \begin{cases} 0 & n < 0 \\ \beta^n & n = 0, 1, 2, 3, 4, 5 \\ 0 & n > 5 \end{cases}$$

and System #2 is described by the difference equation

$$y_2[n] = y_1[n] - \beta y_1[n-1]$$

Determine the impulse response function of the overall cascade system.

(b) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1. Give numerical values of the filter coefficients for the specific case where $\beta = \frac{1}{2}$.

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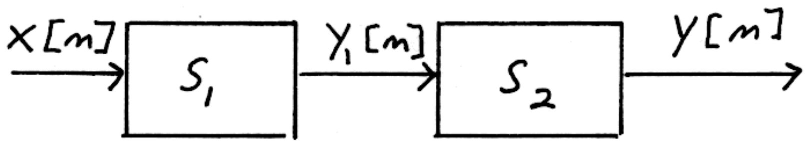
SOLUTION

CASCADE CONNECTION



$$\#1 \quad h_1[n] = \begin{cases} 0 & n<0 \\ \beta^n & n = 0\ 1\ 2\ 3\ 4\ 5 \\ 0 & n>5 \end{cases}$$

$$\#2 \quad h_2[n]:$$

$$y_2[n] = y_1[n] - \beta y_1[n-1]$$

$$h_2[n] = \delta[n] - \beta\,\delta[n-1]$$

The cascade impulse response is determined by the convolution $h_1 * h_2$

n	0	1	2	3	4	5	6
$h_1[n]$	1	β	β^2	β^3	β^4	β^5	0
$h_2[n]$	1	$-\beta$	0	0	0	0	0
$h_2[0]h_1[n]$	1	β	β^2	β^3	β^4	β^5	0
$h_2[1]\,h_1[n-1]$	0	$-\beta$	$-\beta^2$	$-\beta^3$	$-\beta^4$	$-\beta^5$	$-\beta^6$
$h_2[2]h_1[n-2]$	0	0	0	0	0	0	0
	$\vdots$						
$y[n]$	1	0	0	0	0	0	$-\beta^6$

overall response: $h[n] = \delta[n] - \beta^6\delta[n-6]$

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(b) Difference Equation for system

$$y[n] = x[n] - \beta^6 x[n-6]$$

$$b_0 = 1$$
$$b_n = 0 \,, \quad n \neq 0 \text{ or } 6$$
$$b_6 = -\beta^6$$

$$\beta = 1/2$$
$$b_6 = -\frac{1}{2^6} = -\frac{1}{64}$$

$$y[n] = x[n] - \frac{1}{64} x[n-6]$$

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Alternate Approach

(b) $$y_1[n] = \sum_{k=0}^{M} b_k x[n-k]$$

b_k are the impulse response coefficients

$$b_k = \begin{cases} 0 & k<0 \\ \beta^k & k=0,1,2,3,4,5 \\ 0 & k>5 \end{cases}$$

$$y_2[n] = y_1[n] - \beta y_1[n-K]$$

$$y_2[n] = \sum_{k=0}^{M} b_k x[n-k] - \beta \sum_{k=0}^{M} b_k x[n-1-k]$$

$M = 5$, $b_k = \beta^k$

$$y_2[n] = \sum_{k=0}^{5} \beta^k x[n-k] - \beta \sum_{k=0}^{5} \beta^k x[n-1-k]$$

$$y_2[n] = y[n] = x[n] - \beta^6 x[n-6]$$

if $\beta = 1/2$

$$y[n] = x[n] - \frac{1}{64} x[n-6]$$

Note: Equation $y[n] = x[n] - \beta^6 x[n-6]$ also follows from inspection of impulse response $h[n] = \delta[n] - \beta^6 \delta[n-6]$ [previous page]

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