PROBLEM:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = e^{x[n]}$

(b) $y[n] = x[n]\cos(0.2\pi n)$

(c) $y[n] = -x[n+1] + x[n] - x[n-1]$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

a) $y[n] = e^{x[n]}$ $\quad e^{(x_1[n]+x_2[n])} \neq e^{x_1[n]} + e^{x_2[n]}$

not additive

$e^{cx[n]} \neq c e^{x[n]}$

not homogeneous

$\boxed{\therefore \text{ not linear}}$

$T\{x[n-N]\} = e^{x[n-N]} = y[n-N]$

$\boxed{\therefore \text{ Time invariant}}$

$\boxed{\text{Causal}}$ because $y[n]$ never precedes $x[n]$

b) $y[n] = x[n]\cos(.2\pi n)$

$T\{x_1[n] + x_2[n]\} = (x_1[n] + x_2[n])\cos(.2\pi n)$

$= T\{x_1[n]\} + T\{x_2[n]\} \Rightarrow$ additive

$T\{c\,x[n]\} = c\,x[n]\cos(.2\pi n) = c\,T\,x[n] \Rightarrow$ homogeneous

$\boxed{\therefore \text{ linear}}$

$\boxed{\text{Not time invariant}}$

Why? Because

$y[n-N] = x[n-N]\cos(.2\pi(n-N))$

$T\{x[n-N]\} = x[n-N]\cos(.2\pi n)$

$y[n-N] \neq T\{x[n-N]\}$

$\boxed{\text{Causal}}$ $y[n]$ never precedes $x[n]$

c) $y[n] = -x[n+1] + x[n] - x[n-1]$

$T\{x_1[n] + x_2[n]\} = -x_1[n+1] - x_2[n+1] + x_1[n] + x_2[n] - x_1[n-1] - x_2[n-1]$

$= T\{x_1[n]\} + T\{x_2[n]\}$ additive

$T\{c\,x[n]\} = -c\,x[n+1] + c\,x[n] - c\,x[n-1] = c\,T\{x[n]\}$

homogeneous

$\boxed{\therefore \text{ linear}}$

$\boxed{\text{shift invariant}}$

$y[n-N] = -x[n+1-N] + x[n+N] - x[n-1-N]$

$T\{x[n-N]\} = -x[n+1-N] + x[n+N] - x[n-1-N]$

equal

$\boxed{\text{Not causal}}$ $y[n]$ precedes $x[n]$. To illustrate observe $y[n]$ at $n=0$

$y[0] = -x[1] + x[0] - x[-1]$

$x[1]$ is the sample value at $n=1$ $\boxed{\begin{array}{c} n=1 \\ \text{precedes} \\ n=0 \end{array}}$