

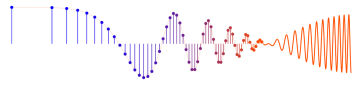
PROBLEM:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = e^{x[n]}$

(b) $y[n] = x[n] \cos(0.2\pi n)$

(c) $y[n] = -x[n + 1] + x[n] - x[n - 1]$



a) $y[n] = e^{x[n]}$

$$e^{(x_1[n] + x_2[n])} \neq e^{x_1[n]} + e^{x_2[n]}$$

not additive

$$e^{cx[n]} \neq ce^{x[n]}$$

not homogeneous

\therefore not linear

$$T\{x[n-N]\} = e^{x[n-N]} = y[n-N]$$

\therefore Time invariant

Causal because $y[n]$ never precedes $x[n]$

b) $y[n] = x[n] \cos(2\pi n)$

$$T\{x_1[n] + x_2[n]\} = (x_1[n] + x_2[n]) \cos(2\pi n) = T\{x_1[n]\} + T\{x_2[n]\} \Rightarrow \text{additive}$$

$$T\{cx[n]\} = cx[n] \cos(2\pi n) = cT\{x[n]\} \Rightarrow \text{homogeneous}$$

\therefore linear

Not time invariant

Why? Because

$$y[n-N] = x[n-N] \cos(2\pi(n-N))$$

$$T\{x[n-N]\} = x[n-N] \cos(2\pi n)$$

$$y[n-N] \neq T\{x[n-N]\}$$

Causal $y[n]$ never precedes $x[n]$

c) $y[n] = -x[n+1] + x[n] - x[n-1]$

$$T\{x_1[n] + x_2[n]\} = -x_1[n+1] - x_2[n+1] + x_1[n] + x_2[n] - x_1[n-1] - x_2[n-1] = T\{x_1[n]\} + T\{x_2[n]\} \Rightarrow \text{additive}$$

$$T\{cx[n]\} = -cx[n+1] + cx[n] - cx[n-1] = cT\{x[n]\} \Rightarrow \text{homogeneous}$$

\therefore linear

Shift invariant

$$y[n-N] = -x[n+1-N] + x[n-N] - x[n-1-N]$$

$$T\{x[n-N]\} = -x[n+1-N] + x[n-N] - x[n-1-N]$$

↗ equal

Not causal

$y[n]$ precedes $x[n]$. To illustrate observe $y[n]$ at $n=0$

$$y[0] = -x[1] + x[0] - x[-1]$$

$x[1]$ is the sample value at $n=1$

$n=1$ precedes $n=0$