The Role of Calculators in Math Education

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Preface

This document outlines the benefits of calculator use in mathematics classrooms from Kindergarten through the University level. Calculators are tools for doing mathematical computations. This document describes how calculators, when used appropriately, can also be a tool for *learning* mathematics. Appropriate use of calculators is a way of increasing the amount and the quality of learning afforded students during the course of their mathematics education.

This document is presented in a modular format – Each section may stand alone or be read as a part of the larger paper. Some of the modules apply only to students of certain grade levels, while others pertain to students of all ages. You can read only the sections that interest you. Research from cited studies is presented in endnotes following each of the last three modules.

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Introduction

Since its invention over thirty years ago, the electronic calculator has evolved from a machine that could only perform simple four-function operations (addition, subtraction, multiplication, division) into one that can now also execute highly-technical algebraic symbolic manipulations instantly and accurately. Each new generation of calculators builds on the previous one with heightened speed and more advanced capabilities. At the same time, the cost of a basic calculator has dropped so low that virtually every household in the United States can afford at least one.

Calculators allow students access to mathematical concepts and experiences from which they were previously limited with only paper and pencil. Because calculators make possible mathematical exploration, experimentation, and enhancement of learning mathematical concepts, the National Council of Teachers of Mathematics (NCTM) and various other organizations and individuals recommend that appropriate calculators be made available for use by students at every grade level from kindergarten through college. Despite the extensive research documenting the benefits of calculator use, there are still many skeptics who worry that calculator use will impair students' mathematical ability and result in increased mathematical illiteracy.

The reality, however, is that calculators are valuable educational tools that allow students to reach a higher level of mathematical power and understanding. By reducing the time that, in the past, was spent on learning and performing tedious paper-and-pencil arithmetic and algebraic algorithms, calculator use today allows students and teachers to spend more time developing mathematical understanding, reasoning, number sense, and applications. Four-function, scientific, and graphing calculators, as well as calculators with computer symbolic algebra manipulation capability provide new pedagogical enhancement opportunities. They afford students learning tools that complement, but do not replace, mental and paper-and-pencil skills, and they expand students' ability to solve problems by providing multiple solution techniques.

Rote computations and tedious algebraic manipulations have historically turned many students away from mathematics. The subject of mathematics has traditionally been thought of as memorizing formulas and substituting numbers in equations, drilling endlessly, and performing long, monotonous computations. The students who could perform these manipulations and computations quickly and accurately were considered to be mathematically inclined; those who were turned off by the mechanical operations were thought to be poor math students. Calculator technology allows students who would ordinarily be frustrated or bored by these tedious manipulations to have access to the real mathematics itself, thus gaining a higher level of mathematical understanding, rather than giving up. The fact is, calculators are better tools to do some of the computations and manipulations that were once done with paper and pencil. In the past, paper and pencil were the only tools available. Appropriate use of technology and associated pedagogy will get more students thinking and reasoning mathematically. Thus more people will develop useful mathematical understanding and mathematical power.

Calculators now come in a number of sizes and styles, and they cover a tremendous range of capabilities, functions, and prices. Despite the myths of harmful consequences resulting from their use, calculators are a pedagogical tool of great value. Teachers of different grade levels and the general public harbor varying preconceived beliefs as far as the use of calculators in the classroom is concerned. Fears regarding the ill effects of calculator use, however, are unfounded. Research has proven that calculators are beneficial to students at every level of education. (Specific references are located in footnotes at the end of the last three modules.) Calculators serve as an equalizer in mathematics education. Not only do they allow students who would ordinarily be turned off by traditional mathematics' tedious computations and algorithms to experience true mathematics, but they also help students to more quickly and readily develop number sense, gain mathematical insight and reasoning skills, value mathematics, and cultivate mathematical understanding, while they enjoy what they are learning.

Dispelling the Myths

Greatly impeding the universal acceptance of calculators into classrooms are the myths that exist regarding calculator use. These myths only serve to slow the inevitable implementation of technology in classrooms and put students at a disadvantage in a world that is rapidly embracing technology. Evidence from research has proven calculators to be effective learning tools; yet, because of the circulation of misinformation with regard to their use, many people continue to believe they are harmful. It is important that these myths are addressed, so calculators can be appropriately incorporated into curricula from kindergarten through the university level.

Myth #1: Calculators are a crutch: They are used because students are too lazy to compute the answers on their own; they do the work for the student. There is almost no mathematical thinking involved in doing rote computations. A real comprehension of mathematics comes as a result of understanding what the question is asking, knowing how to set up the problem, deciding which operations are appropriate, and determining whether or not the answer obtained makes sense. Calculators are simply a tool students use to help solve problems. Since they eliminate tedious computations and algebraic manipulations that discourage many students, calculators allow more students to solve problems and appreciate the power and value of mathematics in the world today. When used appropriately, calculators *enhance* learning and thinking, they do not replace it.

Myth #2: Because calculators do all of the work for the student, he/she will not be stimulated or challenged enough. Calculators do only the low level tasks of computation - they do not "think." Calculators can speed up the learning process. Students understanding the appropriate use of calculators experience more time to explore challenging and interesting mathematics. Calculators permit students to work enough problems to discover and observe patterns in mathematics, which were seldom seen when computations were done by tedious paper and pencil methods. Students will also be able to focus on useful, practical applications for the theories and concepts they learn in class. In the past, many students in mathematics were doing little more than memorizing rules and formulas; they were doing little thinking, problem solving, or reasoning. With appropriate use of calculators, many more students will have the opportunity to get past the mechanics of computation and manipulation and learn about the true meaning and value of mathematics.

Myth #3: "If I didn't need to use technology to learn math, then neither does my child. After all, I turned out just fine." Because the calculator technology that exists today was not present a generation ago, all computations had to be worked out with pencil and paper in a series of long, tedious steps. The world has rapidly moved in the direction of technology, however, and technology has rendered obsolete many of the techniques and methods that were used previously. Because of technology, more students are now able to explore territories in mathematics that are still uncharted, and they are able to do 'real' mathematics and understand its meaning and value. Often the parents who argue against the use of technology in mathematics classes simply have a fear of the unknown. They remember mathematics as consisting of drills, algorithms, and paper-andpencil manipulations. However, calculators have eliminated the need for great skill in paper-and-pencil arithmetic computation and algebraic manipulation – items that used to be the core of a "proper" mathematics education. Computations that took several minutes and many sheets of notebook paper before can now be executed with the touch of a button. Since technology is being implemented in classrooms all over the world, all students must begin to understand technology and its appropriate use now, in order to learn the new technology core skills that will be necessary in the future.

Myth #4: The use of calculators prevents students from effectively learning the basic mathematics they will need when they enter the workforce. Calculators facilitate the mathematics learning process by eliminating tedious and needless paperand-pencil calculations. However, they also familiarize students with technology, increasing their comfort level with technology and giving them a competitive advantage over those who have never been exposed to technology. Moreover, this understanding of the benefits and limitations of technology, as well as a general knowledge of how it operates, will increase openness and willingness to use new forms of technology. Employers want employees who can think, work cooperatively, solve problems using the most effective methods (technology when appropriate), and communicate "solutions" effectively.

Myth #5: People will become so dependent on calculators that they will be rendered helpless without one. (e.g.: What if the battery dies or the student has to perform a computation when no calculator is available?) It is very important that mental calculations as well as estimation and some paper-and-pencil skills continue to be taught in schools, when those are the most appropriate methods for solving problems. Such skills are necessary in the mathematical learning process. These skills will also come in handy when a calculator is not available and when it is necessary to determine the appropriateness of a calculator result. The fact is, calculators are more efficient and accurate at performing many computations, and they are inexpensive and portable enough to keep in one's purse, pocket, car, or office.

Despite all of their benefits and capabilities, calculators will never be able to replace the human mind when it comes to knowing how to read and understand a problem situation, writing an appropriate equation for the problem, choosing which operations to use to solve the problem, correctly interpreting the solution displayed on the calculator, and determining the appropriateness of the answer. Calculators are only as effective as the information students enter into them. Calculators, in conjunction with mental, paper-and-pencil, and estimation skills when appropriate, comprise the tools to help students work through the computations and manipulations necessary for solving problems. Calculators are like computer word processors to English students. Computer word processors do not "create" essays but they do considerably facilitate the creation of an essay. Calculators do not "understand" mathematics but they do considerably facilitate the understanding of mathematics. Despite all of their capabilities, however, they will never replace the important, complex thought processes of which only humans are capable.

Parents, Calculators, and Kids: What Parents Should Know About the Benefits of Calculator Use

Many parents and even teachers are more than a little apprehensive at the mere thought of implementing calculators into mathematics classrooms at all grade levels. They fear that students will not learn the basics of mathematics, and they worry that calculator button-pushing is the only mathematical skill their children will have acquired upon completion of their mathematics education. Many are also concerned that students will become dependent upon the use of a calculator to the point of not being able to execute simple computations in their daily lives without the aid of a calculator. These fears are understandable: the incorporation of new technology often carries with it skepticism and fear of the unknown. After all, the mathematics that most adults remember consisted primarily of performing long, tedious computations and algebraic manipulations using either paper and pencil or tables, memorizing formulas, and endlessly drilling the skills they had learned.

Mathematics has grown substantially in the last fifty years, and the tools available to aid mathematics students have also changed dramatically. Mathematics today is a subject that is more important for all students to really understand and appreciate; it is a subject that holds more meaning for students, one that allows them to think logically, develop number sense, and cultivate a true mathematical understanding. It is also becoming more interesting and engaging for students. The implementation of calculators and computer technology into mathematics curricula at every level of education is vital to the advancement of mathematical teaching and learning. Calculators and computers, such as hand-held graphing calculators, are not to be feared; they are powerful learning tools that allow students to experience the richness and value of mathematics by greatly reducing the need to execute paper-and-pencil computations and algebraic manipulations.

Mathematics is not about rote computations, memorization, endless drills, or tedious manipulations. Nor is it meant to consist solely of learning and performing algorithms. At its heart, mathematics embodies logic and reasoning, problem solving and number sense, and a search for order. It involves discovery, pattern recognition, applications, concepts, construction of relationships, reasoning from data, problem solving, and abstract thinking. Computations and algebraic manipulations are merely a means by which one gets to the mathematics; they are not an end within themselves. By instantly performing operations that would otherwise take several minutes or even hours, calculators free up valuable class time so that it can be spent on learning higher order thinking and reasoning skillsⁱ.

Part of the uneasiness felt about the use of calculators in classrooms is a result of the belief that mathematics is and should be hard work, work that is normally associated with manual computations and manipulations. Calculators can eliminate much of that work, making them seem subversive. This leads to a stigma and causes negative words such as "crutch," "laziness," and "cop-out" to be associated with calculator use. Ironically, computers that perform the same functions as calculators are considered to be advanced technological tools that are welcomed into classrooms. In truth, calculators are nothing more than well-programmed, application specific, miniature computers. They offer many of the conveniences of a computer; yet they are small enough to carry in a pocket, purse, or backpack. Some are even simple enough to be operated by a preschooler. Moreover, they are inexpensive enough to purchase for home use or as a classroom set. Graphing calculators, which bring the power of computer visualization to all students, can cost less than a pair of trendy new tennis shoes – an entire class set can be purchased for approximately the same amount of money it would cost to buy one fully equipped computer, which would only serve two students at a time.

Calculators simplify tasks, but they do *not* do the real work for the student. It is still up to the student to read the problem, understand what is asked, determine an appropriate mathematical model (e.g. an equation to solve), solve the equation using the most appropriate method (mental, paper and pencil, or technology), interpret the answer in the problem situation to determine whether the answer makes sense. Calculators enable students to focus more on the "whys" of mathematics than on the "hows." And they facilitate the formulation and testing of conjectures and the verification of solutions, since students' questions and suppositions can now be answered almost instantly. Mathematical investigations that were not possible just a few years ago are now explored with ease by students, opening the door for them to make more complex and insightful discoveries. The use of calculators simply allows teachers and students to spend more time on the non-computational parts of the problem-solving process, since the real mathematics is not found in the low-level manipulative procedures anywayⁱⁱ.

Calculators are able to perform arithmetic operations and algebraic manipulations with much greater speed and accuracy than can humans. If the technology is available, why not use it when it is the most appropriate method for solving the problem? No school student uses quills for writing school reports today; they use computers with word-processing software. No one uses slide rules today because calculators are better tools for the job. Research has proven that using calculators does not harm students' algebraic skills or procedural knowledgeⁱⁱⁱ and that students who use them often demonstrate a better understanding of concepts than do their non-calculator-using counterparts^{iv}. It is also easier for students to solve problems when they can focus on the problem-solving process rather than on rote computations and symbolic-manipulation algorithms.

In addition to speeding up computations and allowing for more class time to be spent on learning real mathematics, calculators serve several other important purposes. Several investigations and studies have proven that calculator use makes students more confident about their mathematical ability and the work they are doing, relieves mathematics anxiety, increases persistence and enthusiasm, and improves students' attitudes toward mathematics^v. Just knowing that they have a calculator to verify their answers improves students' performance in problem solving and generates more positive feelings about the activity at hand. Furthermore, the use of *graphing* calculators has also been shown to close the gender gap, with female students performing as well as male students in most areas when graphing calculators are incorporated into the curriculum^{vi}.

Calculators allow students to become more active learners. Rather than just sitting back and passively accepting the examples given by the teacher, students are encouraged to develop their own examples and formulate their own hypotheses. Students who were once thought to be poor mathematicians because they were turned off or disabled by tedious, time-consuming computations may now have the opportunity to prove themselves superior when it comes to understanding and problem solving. Calculator use also promotes group work and communication among peers and classmates and, subsequently, the exchange of ideas and conjectures.

While it is understandable that parents and some teachers are apprehensive about implementing calculators into mathematics curricula at all grade levels, it is vital that they look at the facts. There is a myriad of research supporting the use of calculators in mathematics classrooms, and the benefits of calculator-use are extensive. As soon as people over come their fear of technology and begin and to accept that technology is changing the way mathematical computations and manipulations are executed, mathematics instruction will begin to take tremendous strides in its push toward a future of new and better opportunities for student learning.

^{iv} "Research from over 100 studies indicates that the use of calculators (a) promotes achievement, (b) improves problem-solving skills, and (c) increases understanding of mathematical ideas" (Suydam, 1987).

ⁱ Research has shown that calculators can aid in "stimulating problem solving, in widening children's number sense, and in strengthening understanding of arithmetic operations." They can also help students learn basics, such as numbers, counting, and the meaning of arithmetic operations (Campbell & Stewart, 1993).

ⁱⁱ Students also show greater ease in problem-solving when using calculators, since they focus less on computational recall and algorithmic routines and more on the other parts of the problem-solving process. Appropriate calculator use also "promotes enthusiasm and confidence while fostering greater persistence in problem-solving" (Campbell & Stewart, 1993).

ⁱⁱⁱ Children often learn better and retain more information when they use calculators. And frequently students who use calculators rather than paper and pencil for much of their class work score higher on paper-and-pencil tests than do their non-calculator-using counterparts. Studies have also shown that some students learn basic computational facts better through the use calculators, even when learning the basic facts was not the specific reason for using the calculators (Suydam, 1987).

^v "Students using calculators possess a better attitude toward mathematics and an especially better selfconcept in mathematics than noncalculator students. This statement applies across all grades and ability levels." (Hembree and Dessart, 1986)

^{vi} When graphing calculators are incorporated, female performance improves in the areas of confidence, spatial ability, algebra skills, and classroom environment (Dunham, 1995)

Calculators: Elementary School Teachers' Concerns

Implementing calculators into mathematics curricula at the elementary level will allow students to learn more quickly and efficiently while keeping them engaged in what they are learning. By reducing the emphasis on learning computational algorithms, more time will be available to spend on sharpening problem-solving, mental arithmetic, and estimation skills. The implementation of calculators has changed the nature of the problems that are important in mathematics and has opened the door for new methods of investigating those problems. Since calculators reduce the amount of time required to solve problems and learn certain skills, more applications can be considered, and students who were previously turned off by tedious computations may now be more inspired to explore the richness of math.

Calculators can be quite effective aids when it comes to problem solving, reinforcement of computational skills, pattern recognition, and number sense. They can also help teach topics such as percents and fractions, integers, perimeter and area, and exponents. In order to implement calculators as an integral part of kindergarten through sixth grade mathematics classes, appropriate changes will need to be made to the curricula. Less time will need to be spent on tedious paper-and-pencil computations and algebraic manipulations, which means more time will be available for concept development, problem-solving, mental arithmetic, and estimation. New National Council of Teachers of Mathematics (NCTM) Standards-based curricula will also most likely include the earlier introduction of certain new skills and content topics, such as geometry and data analysis and connecting mathematics and science.

The NCTM Curriculum and Evaluation Standards recommends that all students use calculators to:

- explore and experiment with mathematical ideas such as patterns, numerical and algebraic properties, and functions;
- develop and reinforce skills such as estimation, computation, graphing, and analyzing data;
- focus on developing problem-solving processes rather than the computations associated with the problems;
- perform tedious computations that often develop when working with real data in problem situations;

• gain access to mathematical ideas and experiences that go beyond those levels limited by traditional paper-and-pencil computation. (NCTM, 1991)

NCTM further recommends that every teacher at every level promote the use of calculators to enhance mathematics instruction by:

- modeling the use of calculators in a variety of situations;
- using calculators in computation, problem solving, concept development, pattern recognition, data analysis, and graphing;
- incorporating the use of calculators in testing mathematical skills and concepts;
- keeping current with the state-of-the-art technology appropriate for the grade levels being taught;
- exploring and developing new ways to use calculators to support instruction and assessment (NCTM, 1991)

Developing paper-and-pencil as well as mental computation ability is a valid concern of teachers. Research, however, has suggested that introduction of calculators as early as preschoolⁱ does not harm computational ability. In fact, studies have shown that appropriate use of calculators enhances young children's ability to learn basic factsⁱⁱ and that students who use calculators frequently exhibit more advanced concept development and problem-solving skills than those who do not use calculatorsⁱⁱⁱ. In addition, students who use calculators exhibit greater self-confidence and achievement, and calculator use generates more enthusiasm about mathematics^{iv}. Using calculators in the classroom also promotes peer cooperation, communication, exchange of ideas, and an increased level of comfort with technology. And calculators enable students of a wider range of abilities to function in the same classroom. In addition to all of these benefits, many calculators are inexpensive and easily accessible, and four-function models are simple enough to be mastered by small children.

Calculator use can help students focus on the more important aspects of the problem-solving process. They can have more time to spend on reading the problem and making sure they understand what it is asking, setting up the problem, correctly reading the display, and determining whether or not the answer is reasonable^v. Students can investigate their own approaches to problem solving, making their own conjectures, and

testing them out on the calculator to quickly see if they were correct^{vi}. They can develop their own examples, giving them a sense of ownership and making them feel that it is truly their work and not just that of the teacher.

ⁱⁱⁱ According to the collective findings of 79 research reports, at every grade level from kindergarten through 12 (except grade 4), the use of calculators can improve the average student's paper-and-pencil skills, both in basic operations and in problem solving. The use of calculators on examinations produced much higher achievement scores than paper-and-pencil alone, in both basic operations and problem solving. This applies across all grade and ability levels (Hembree and Dessart, 1986). In addition, the results from the Third International Mathematics and Science Study, conducted in 1995, showed that U.S. fourth graders scored above the international average in mathematics (NCTM, 1997).

^{iv} "Students using calculators possess a better attitude toward mathematics and an especially better selfconcept in mathematics than noncalculator students. This statement applies across all grade and ability levels" (Hembree and Dessart, 1986).

^v Students also show greater ease in problem-solving when using calculators, since they focus less on computational recall and algorithmic routines and more on the other parts of the problem-solving process. The appropriate use of calculators has also be shown to result in greater persistence in problem solving (Campbell & Stewart, 1993).

^{vi} When calculators are incorporated into the learning process, achievement in problem solving increases, and more solution methods and strategies are utilized. Moreover, "the calculator makes exploration of hypotheses feasible, and is useful in developing counting, computation, estimation, and other mathematical skills" (Suydam, 1985).

ⁱ "The evidence indicates that they can start using calculators when they enter school: calculators can be used as an aid to counting" (Suydam, 1982)

ⁱⁱ "No evidence was found that elementary school students become calculator-dependent... However, students were less afraid to attempt difficult problems when they could use calculators" (Suydam, 1982).

Graphing Calculators: Issues Affecting Secondary School Teachers and University Professors

When students work with graphing calculators, they have the potential to work much more intelligently than they could if they were not using this valuable resource; they form an "intelligent partnership" with the graphing calculator (Jones, 1996). Research has shown that graphing calculators can improve classroom dynamics, boost students' confidence levels, and promote the understanding of mathematical concepts and functions, and advance problem-solving abilityⁱ. That is not to say that mental arithmetic and paper-and-pencil and estimation skills are no longer valuable – they are. But there must be a balance. It is important to use the tool that is most appropriate for the job. When they are used appropriately, graphing calculators do not pose a threat to students' ability to perform algebraic manipulations or proceduresⁱⁱ.

Graphing calculators help students visualize problems, discover mathematical theorems on their own, instantly check the validity of their answers, test out their own hypotheses, and explore different ways of solving problemsⁱⁱⁱ. Graphing calculators allow topics to be discovered by students on their own, even before they are formally introduced by the teacher. They facilitate an active approach to learning, converting a classroom from a place where students sit back passively listening to the instructor, to one where students work with their classmates and produce their own ideas and solutions^{iv}. Graphing calculators improve communication among students, and they allow students a faster, better way to produce graphs^v; this is a much more efficient, accurate method than drawing graphs by paper and pencil alone.

Computer graphing (e.g. using graphing calculators) provides an important new teaching and learning paradigm: Graphs can now be used *to study math*. In the past, students studied advanced mathematics (calculus) to learn how to draw graphs accurately. Now computer-generated graphs can be used to study important mathematical concepts. Inexpensive graphing calculators make the power of computer visualization a reality for all secondary and college students.

Calculators offer students a method of performing computations and algebraic manipulations that is more efficient and precise than paper-and-pencil methods alone. Graphing calculators can store, manipulate, and display data in many different ways. They can be thought of as well-programmed miniature special purpose computers that are very portable, much more affordable, and easier to use than desktop computers. They are inexpensive enough so that classrooms may have a set for students to use at any time, and many students will be able to afford their own. They cost about as much as a pair of trendy new athletic shoes, and a class set can be purchased for as much as it would cost to buy just one or two computers. Each generation of graphing calculators is becoming more powerful and useful; yet they are not getting more difficult to use.

Graphing calculator technology is changing the types of problems that are important in mathematics classrooms. When algebraic manipulations were the most important concept to be learned, they naturally comprised the focus of most teaching. Now, however, arithmetic calculations and algebraic manipulations can be performed correctly in a matter of seconds using technology. Students can avoid time-consuming, tedious procedures and concentrate on understanding concepts, developing higher order thinking skills, and learning relevant applications. These powerful calculators provide students with another resource -- in addition to paper-and-pencil, mental, and estimation skills – to assist them in executing the procedures necessary to understand and apply mathematics.

Calculators allow for the cultivation of analytical adeptness and proficiency in complex thought processes, rather than just the development of mechanical, computational skills. Problems representing "real-life" situations with complicated numbers can also be addressed. And students will not have to worry about whether they are going to make a mistake in addition or multiplication and not come up with the correct answer, even though they worked the rest of the problem correctly. Students still need to be taught to develop good mental estimation skills, so they will know if the calculator solution makes sense in the problem situation. Use of calculators allows students to focus on the steps involved in problem solving. This is just as important, if not more so, than the actual answer. Students value what is tested. Therefore, since learning how to use technology appropriately is so very important today, calculators should also be permitted on examinations. If students are allowed to use calculators in class and for homework, it would not be appropriate to deprive them of this valuable resource on tests. That is not to say that calculators must be allowed on every portion of every examination. For example, the College Board's Advanced Placement Calculus exam has two parts: in one part graphing calculators are required, while in the other portion, calculators are forbidden.

The use of graphing calculators in the classroom extends problem-solving and mathematical understanding by making both practical and possible an important learning theory called *multiple linked representations*. Mathematical concepts can now be experienced through, and problems solved by, numerical, graphical, and symbolic representations. Graphing calculators allow students to move easily between these representations^{vi}. When students are able to choose between several methods of solving a problem, it is more likely that they will remember how to solve it and be able to solve a similar type of problem the next time they see one. Moreover, seeing the graphs helps reinforce abstract concepts and allows the problem to become more tangible. Students can also use graphical methods to confirm an answer obtained through algebraic methods. Research has also shown that access to graphical representations has improved females' ability to visualize functions and graphs.^{vii}

When students are able to spend more time concentrating on understanding, setting up, and choosing the appropriate operations and equations for a problem, they tend to see mathematics as more useful and less tedious than those who are forced to perform all algebraic manipulations by paper and pencil alone. They are more likely to stay with the problem, and their confidence about their mathematical abilities is boosted. Research not only proves that the use of calculators results in more positive feelings and better attitudes about mathematics for both students and teachers^{viii}, but it also confirms that calculators improve performance in a variety of areas including problem solving (Dunham, 1995).

ⁱ "In almost all cases, students taught with calculators (but tested without technology) had achievement scores for computation as high or higher than those taught without technology. With calculators, students had higher problem-solving scores, better attitudes toward mathematics, and better self-concepts of their own ability to do mathematics. Recent studies suggest that graphing calculators and computer symbolic algebra systems can be just as beneficial to student learning" (Dunham, 1993).

ⁱⁱ There are more studies that show a positive benefit from graphing instruction than there are studies that show 'no significant difference' – "especially in the areas of problem solving, spatial visualization, precalculus achievement, and calculus readiness. If paper-and-pencil skills are taught in conjunction with graphing methods, studies show no loss of skills even when students are tested without technology" (Questions and Answers About Using Hand-Held Technology to Teach Mathematics, 1997).

ⁱⁱⁱ "Dunham's review of research (1993) reports that many students who use graphing technology:

*place at higher levels in hierarchy of graphical understanding;

*are better able to relate graphs to their equations;

*can better read and interpret graphical information;

*obtain more information from graphs;

*have greater overall achievement on graphing items;

*are better at "symbolizing," that is, finding an algebraic representation for a graph;

*better understand global features of functions;

*increase their "example base" for functions by examining a greater variety of representations; and *better understand connections among graphical, numerical, and algebraic representations" Moreover, they:

"*had more flexible approaches to problem solving;

*were more willing to engage in problem solving and stayed with it longer;

*concentrated on the mathematics of the problem and not on the algebraic manipulation;

*solved nonroutine problems inaccessible by algebraic techniques; and

*believed calculators improved their ability to solve problems" (Dunham & Dick, 1994).

^{iv} In studies where graphing technology was in use, students were more active, and they participated in more group work, investigations, problem solving, and explorations. Teachers lectured less and were often used by students as more of a consultant than a task-setter (Dunham, 1993; Dunham & Dick, 1994).

^v "Students who use graphing calculators are better able to read and interpret graphs, understand global features of graphs, relate graphs to their equations, and make connections among multiple representations of functions" (Dunham, 1996).

^{vi} Because graphing calculators allow students to move freely from numerical to graphical to symbolic representations, each student can approach problems using different representations. Being able to use the method which best suits the student causes the student to perform better and become more confident (Quesada, 1996).

^{vii} When graphing calculators are incorporated, female performance improves in the areas of confidence, spatial ability, algebra skills, and classroom environment. The ability to check their algebra and confirm their solutions has inspired confidence in females (Dunham, 1995).

^{viii} Graphics calculators have also been show to improve attitudes of instructors, saying the graphing calculators help motivate students (Almeqdadi, 1997; Suydam, 1982).

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