Finding areas under the normal

A linear function of a normal random variable is also normally distributed. In general if (as in p. 24):

$$x = \mu + \epsilon \qquad \epsilon \sim N(0, \sigma^2)$$

then,

$$x - \mu = \epsilon \qquad \Rightarrow x - \mu \sim N(0, \sigma^2)$$

and finally,

$$\frac{x-\mu}{\sigma} = \frac{\epsilon}{\sigma} \qquad \Rightarrow \frac{x-\mu}{\sigma} \sim N(0,1)$$

which represents deviations from mean in units of standard deviation.

Example

Suppose X is normally distributed with mean 10, and variance 25, that is,

$$X \sim N(10, 25)$$

Then, what is the probability

$$P(12 \leq X \leq 15) = ?$$

Given the mean and the variance Excel calculates this probability using the command `normdist(X,mean,st dev,1)`. In this case the whole command will be:

```
=normdist(15,10,5,1)-normdist(12,10,5,1)
```

Kadd (the Excel add-in) has a `Probability` command that calculates this probability by specifying the mean, the standard deviation, and the interval (12-15).

You can verify this using a table for the stardard normal and calculating the following probability:

$$\begin{aligned}
P(12 \leq X \leq 15) &= P\left(\frac{12-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{15-\mu}{\sigma}\right) \\
&= P\left(\frac{12-10}{5} \leq Z \leq \frac{15-10}{5}\right) \\
&= P(0.4 \leq Z \leq 1) \\
&= P(0 \leq Z \leq 1) - P(0 \leq Z \leq 0.4) \\
&= 0.3413 - 0.1554 \\
&= 0.1859
\end{aligned}$$

which graphically can be represented as follows: