

## EXPLORATORY ANALYSIS ENABLED BY MULTIREOLUTION, MULTIPERSPECTIVE MODELING

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### ABSTRACT

The objective of exploratory analysis is to gain a broad understanding of a problem domain before going into details for particular cases. Its focus is understanding comprehensively the consequences of uncertainty, which requires a good deal more than normal sensitivity analysis. Such analysis is facilitated by multiresolution, multiperspective modeling (MRMPM) structures that are becoming increasingly practical. A knowledge of related design principles can help build interfaces to more normal legacy models, which can also be used for exploration.

### 1 BACKGROUND

Strategy problems are typically characterized by enormous uncertainties that should be central in assessment of alternative courses of action—although individuals and organizations often suppress those uncertainties and give a bizarre level of credence to wishful-thinking planning factors and other simplifications (Davis 1994 Ch. 4, Davis, Gompert, and Kugler 1996). In the past, an excuse for downplaying uncertainty analysis—except for marginal sensitivity analysis around some “best-estimate” baseline of dubious validity—was the sheer difficulty of doing better. The time required for setup, run, and analysis made extensive uncertainty work infeasible. Today, technology permits extensive uncertainty analysis with personal computers.

A key to treating uncertainty well is *exploratory analysis* (Davis and Hillestad 2001). The objectives of exploratory analysis include understanding the implications of uncertainty for the problem at hand and informing the choice of strategy and subsequent modifications. In particular, *exploratory analysis can help identify strategies that are flexible, adaptive, and robust*. In successive sections, this paper describes exploratory analysis; puts it in context; discusses enabling technology and theory; points to companion papers applying the ideas; and concludes with some technology challenges for modeling and simulation. The paper draws heavily on a forthcoming book (Davis and

Hillestad 2001) and builds on a much rougher preliminary presentation of the same material (Davis 2000).

### 2 EXPLORATORY ANALYSIS

#### 2.1 What Exploratory Analysis Is and Is Not

Exploratory analysis examines the consequences of uncertainty. It can be thought of as sensitivity analysis done right, but is so different from usual sensitivity analysis as to deserve a separate name. It is closely related to scenario space analysis (Davis 1994 Ch. 4) and “exploratory modeling” (Banks 1993, Lempert et al. 1996). It is particularly useful for gaining a broad understanding of a problem domain before dipping into details. That, in turn, can greatly assist in the development and choice of strategies. It can also enhance “capabilities-based planning” by clarifying *when*—i.e., in what circumstances and with what assumptions about all the other factors—a given capability such as an improved weapon system or enhanced command and control will likely be sufficient or effective (Davis, Gompert, and Kugler 1996). This contrasts with establishing a base-case scenario, and an organizationally blessed model and data base, and then asking “How does the outcome change if I have more of this capability?”

#### 2.2 Types of Uncertainty

Uncertainty comes in many forms and it is useful (National Research Council 1997) to distinguish between input uncertainties (i.e., parametric uncertainties) and structural uncertainty. Input uncertainty relates to imprecise knowledge of the model’s input values. Structural uncertainty relates to questions about the form of the model itself: Does it reflect all the variables on which the real-world phenomenon purportedly described by the model depends? Is the analytical form correct? Some uncertainties may be inherent because they represent stochastic processes. Some may relate to fuzziness or imprecision, while others reflect discord among experts. Some relate to knowledge about the

values of well-defined parameters, whereas others refer to future values that as yet have no true values.

It is convenient to express the uncertainties parametrically. If unsure about the model's form, we can describe this also to some extent with parameters. For example, parameters may control the relative size of quadratic and exponential terms in an otherwise linear model. Or a discrete parameter may be a switch choosing among distinct analytical forms. Some parameters may apply to the deterministic aspect of a model, others to a stochastic aspect. For example, a model might describe the rate at which Red and Blue suffer attrition in combat according to a simplistic Lanchester square law:

$$\frac{d\tilde{R}}{dt} = -\tilde{K}_b \tilde{B}(t) \quad \frac{d\tilde{B}}{dt} = -\tilde{K}_r \tilde{R}(t)$$

where the attrition coefficients for Red and Blue have both deterministic and stochastic parts, each of which are subject to uncertainty, as in (illustrating for Blue only)

$$\tilde{K}_b(t) = K_{bo}[1 + c_b \tilde{N}_b(t; \mu, \sigma_b)].$$

Here the  $N$  term is a normal random variable with mean  $\mu$  and standard deviation  $\sigma$ . It represents stochastic processes occurring within a particular simulated war, e.g., from one time period to the next. The means and standard deviations are ordinary deterministic parameters, as are the coefficients  $K_{bo}$ ,  $K_{ro}$ ,  $c_r$ , and  $c_b$ . These have constant values within a particular war, but at what value they are constant is uncertain.

So far the equations have represented input uncertainty. However, suppose there is controversy over using the linear, square, or some hybrid version of a Lanchester equation. We could represent this dispute as input, or parametric, uncertainty by modifying the equation to read

$$\frac{d\tilde{R}}{dt} = -\tilde{K}_b \tilde{B}^e(t) \tilde{R}^f(t) \quad \frac{d\tilde{B}}{dt} = -\tilde{K}_r \tilde{B}^g(t) \tilde{R}^h(t).$$

Now, by treating the exponents as uncertain parameters, we could explore both input and structural uncertainties in the model—at least to some extent. The fly in the ointment is that nature's combat equations are much more complex (if they exist), and we don't even know their form. Suppose, merely as an example, that combatant effectiveness decays exponentially as combatants grow weary. We could not explore the consequences of different decay times if we did not even recognize the phenomenon in the equation's form. In fact, we *often* do not know the true system model. Nonetheless, much can be accomplished by allowing for diverse effects parametrically.

### 2.3 Types of Exploratory Analysis

Exploratory analysis can be conducted in several ways (Davis and Hillestad 2001). Although most of the methods have been used in the past (see especially Morgan and Henrion 1992), they are still not appreciated and are often poorly understood.

*Input exploration* (or *parametric exploration*) involves conducting model runs across the space of cases defined by discrete values of the parameters within their plausible domains. It considers not just excursions taken one-at-a-time as in normal sensitivity analysis relative to some presumed base-case set of values, but rather all the cases corresponding to value combinations defined by an experimental design (or a smaller sample). The results of such runs, which may number from dozens to hundreds of thousands or more, can be explored interactively with modern displays. Within perhaps a half-hour, a good analyst doing such exploration can often gain numerous important insights that were previously buried. He can understand not just which variables “matter,” but *when*. For example, he may find that the outcome of the analysis may be rather insensitive to a given parameter for the so-called base case of assumptions, but quite sensitive for other plausible assumptions. That is, he may identify in what cases the parameter is important. To do capabilities-based planning for complex systems, this can be distinctly nontrivial.

A complement to parametric exploration is “*probabilistic exploration*” in which uncertainty about the input parameters is represented by distribution functions representing the totality of one's so-called objective and subjective knowledge. I sometimes use quotes around “probability” because the distributions are seldom true frequencies or rigorous Bayesian probabilities, but rather rough estimates or analytical conveniences.

Using analytical or Monte Carlo methods, the resulting distribution of outcomes can be calculated. This can quickly give a sense for whether uncertainty is particularly important. In contrast to displays of parametric exploration, the output of probabilistic exploration gives little visual weight to improbable cases in which various inputs all have unlikely values simultaneously. Probabilistic exploration can be very useful for a condensed net assessment. Note that this use of probability methods is different from using them to describe the consequences of a stochastic process within a given simulation run. Indeed, one should be cautious about using probabilistic exploration because one can readily confuse variation across an ensemble of possible cases (e.g., different runs of a war simulation) with variation within a single case (e.g., fluctuation from day to day within a single simulated war). Also, an unknown constant parameter for a given simulated war is no longer unknown once the simulation begins and simulation agents representing commanders should perhaps observe and act upon the correct values within a few sim-

ulated time periods. Despite these subtleties, probabilistic exploration can be quite helpful.

The preferred approach treats some uncertainties parametrically and others with uncertainty distributions. That is, it is *hybrid exploration*. It may be appropriate to parameterize a few key variables that are under one's own control (purchases, allocation of resources, and so on), while treating the uncertainty of other variables through uncertainty distributions. One may also want also to parameterize a few variables characterizing the future context in which strategy must operate (e.g., short warning time). There is no general procedure here; instead, the procedure should be tailored to the problem at hand. In any case, the result can be a comprehensible summary of how known classes of uncertainty affect the problem at hand.

Let me give a few examples of what exploratory analysis can look like. Figure 1 mimics a computer screen during a parametric exploration of what is required militarily to defend Kuwait against a future Iraqi invasion by interdicting the attacker's movement with aircraft and missiles (Davis and Carrillo 1997). Each square denotes the outcome of a particular model case (i.e., a specific choice of all the input values). The model being used depends on 10 variables—those on the x, y, and z axes, and seven listed to the side (the z-axis variable is also listed there, redundantly). The outcome of a given simulation is represented by the color (or, in this paper, by the pattern) of a given square. Thus, a white square represents a good case in which the attacker penetrates only a few tens of kilometers before being halted. A black square represents a bad case in which the attacker penetrates deep into the region that contains critical oil facilities. The other patterns represent in-between cases. The number in each square gives the penetration distance in km.

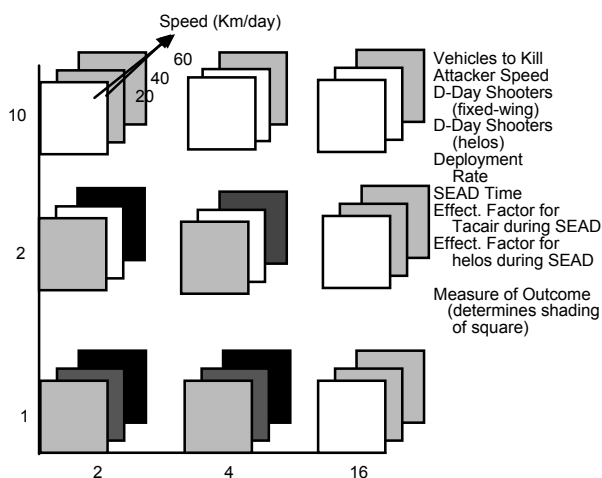


Figure 1: Display of Parametric Exploration

To display results in this way for a sizable scenario space RAND has often used a program called Data View, developed at RAND in the mid 1990s by Stephen Bankes

and James Gillogly. After running the thousands or hundreds of thousands of cases corresponding to an experimental design for parametric exploration, we explore the outcome space at the computer. We can choose interactively which of the parameters to vary along the x, y, and z axes of the display. The other parameters then have the values shown along the right. However, we can click on their values and change them interactively by selecting from the menu of values for cases that have been run.

As mentioned above, in about a half an hour of such interactive work, one can develop a strong sense of how outcomes vary with *combinations* of parameter values. This is much more than traditional sensitivity analysis. Moreover, one can search out and focus upon the “good” cases. Figure 1 is merely one schematic snapshot of the computer screen for choices of parameter values that show some successes. Most snapshots would be dominated by black squares because it is difficult to defend Kuwait against a large threat. Data View is not a commercial product, but RAND has made it available to government clients and some other organizations (e.g., allied military staffs).

Other personal-computer tools can be used for the same purpose and the state of the art for such work is advancing rapidly. A much improved version of Data View called CAR™ is under development by Steve Bankes at Evolving Logic <[www.evolvinglogic.com](http://www.evolvinglogic.com)>. For those who prefer spreadsheet modeling, there are plug-in programs for Microsoft EXCEL® that provide statistical capabilities and some means for exploratory analysis. Two are Crystal Ball® <[www.decisioneering.com](http://www.decisioneering.com)> and @Risk® <[www.palisade.com/.html/risk.html](http://www.palisade.com/.html/risk.html)>. For a number of reasons such as visual modeling and convenient array mathematics, I usually prefer the Analytica® modeling system (the exception is when one needs procedural programming). Analytica <[www.lumina.com](http://www.lumina.com)> is an outgrowth of the Demos system developed at Carnegie Mellon University (Morgan and Henrion, 1992).

Figure 2 shows a screen image from recent work with Analytica on the same problem treated in Figure 1. In this case, we have a more traditional graphical display. Outcome is measured along the Y axis and one of the independent variables is plotted along the X axis. A second variable (D-Day shooters) is reflected in the family of curves. The other independent variables appear in the rotation boxes at the top. As with Data View, we change parameter values by clicking on a value and selecting from a menu of values. Such interactive displays allow us to “fly through the outcome space” for many independent parameters, in this case 9. For this number, the display was still quickly interactive for the given model and computer (a Macintosh PowerBook G3 with 256 MB of RAM).

So far, the examples have focused on parametric exploration. Figure 3 illustrates a hybrid exploration (Davis, et al. 1998). It shows the distribution of simulation outcomes resulting from having varied most parameter

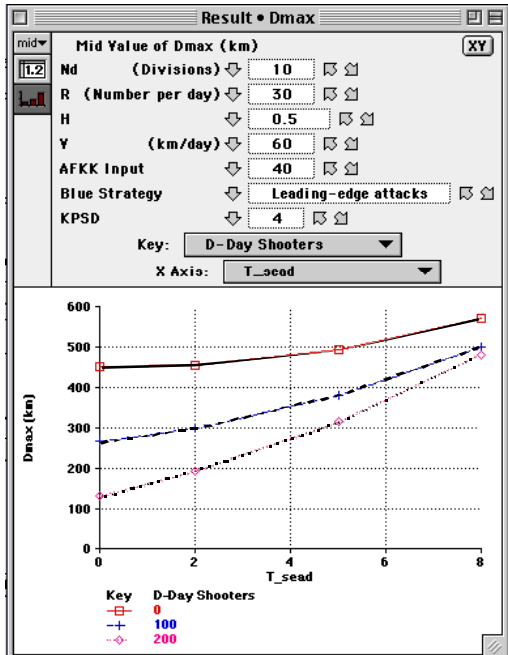


Figure 2: Analytica Display of Parametric Exploration

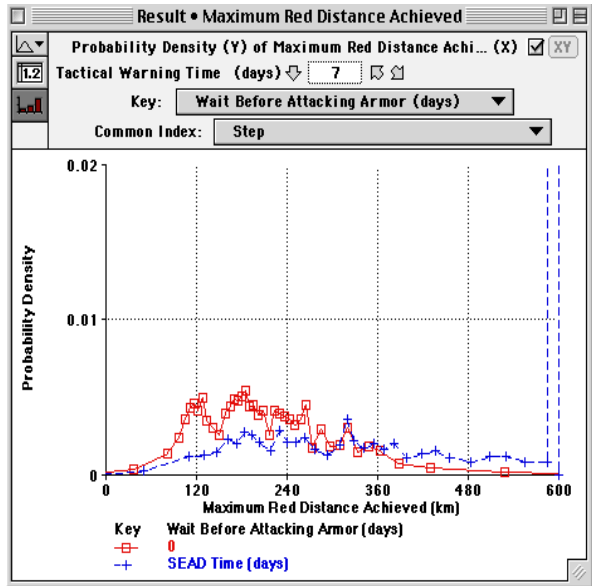


Figure 3: Analytica Display of "Probabilistic" Exploration

values "probabilistically" across an ensemble of possible wars, but with warning time and the delay in attacking armored columns left parametric.

The probabilistic aspect of the calculation assumed, for example, that the enemy's movement rate had a triangular distribution across a particular range of values and that the suppression of air defenses would either be in the range of a few days or more like a week, depending on whether the enemy did or did not have air-defense systems and tactics that were not part of the best estimate. We

represented this possibility with a discrete distribution for the likelihood of such surprises. The two curves in Figure 3 differ in that the one with crosses for markers assumes that interdiction of moving columns waits for suppression of air defenses (SEAD). The other curve assumes that interdiction begins immediately because the aircraft are assumed stealthy.

This depiction of the problem shows how widely the outcomes can vary and how the outcome distribution can be complex. The non-stealthy-aircraft case shows a spike at the right end where cases pile up because, in the simulation, the attacker halts at an objective of about 600km. Note that the mean is not a good metric: the "variance" is huge and the outcome may be multimodal.

These results have been from analyses accomplished in recent years for the Department of Defense. As we look to the future, much more is possible with computational tools. Much better displays are possible for the same information and, even more exciting, computational tools can be used to aid in the search process of exploration. For example, instead of clicking through the regions of the outcome space, tools could automatically find portions of the space in which particular outcomes are found. One could then fine-tune one's insights by clicking around in that much more limited region of the outcome space. Or, if the model is itself driven by the exploration apparatus, then the apparatus could search for outcomes of interest and then focus exploration on those regions of the input space. That is, the experimental design could be an output of the search rather than an input of the analysis process. These methods are at the core of the evolving tool mentioned earlier called CAR (for Computer-Assisted Reasoning).

## 2 EXPLORATORY ANALYSIS IN CONTEXT

Exploratory analysis is an exciting development with a long history with RAND's RSAS and JICM models. However, it is only one part of a sound approach to analysis generally. It is worth pausing to emphasize this point. Figure 4 shows how different types of models and simulations (including human games) have distinct virtues. The figure is specialized to military applications, but a more generic version applies broadly to a wide class of analysis problems.

Type Model	Resolution	Analytical Agility	Decision Breadth	Integration support	Richness of Phenomena	Human actions
Analytical	Low					
Human game	Low					
Campaign	Med.					
Entity-level	High					
Field expt.	High					

Figure 4: Virtues of a Model and Gaming Family

White rectangles indicate “good;” that is, if a cell of the matrix is white, then the type model indicated in the left column is very effective with respect to the attribute indicated in the cell’s column. In particular, analytical models (top left corner), which have low resolution, can be especially powerful with respect to their analytical agility and breadth. In contrast, they are very poor (black cells) with respect to recognizing or dealing with the richness of underlying phenomena, or with the consequences of both human decisions and behavior. In contrast, field experiments often have very high resolution (they may be using the real equipment and people), and may be good or very good for revealing phenomena and reflecting human issues. They are, however, unwieldy and inappropriate for studying issues in breadth. The small insets in some of the cells indicate that the value of the type model for the particular purpose can often be enhanced a notch or two if the models include sensible decision algorithms or knowledge-based models that might be in the form of expert systems or artificial-intelligence agents.

Figure 4 was developed as part of an exhortation to the Department of Defense regarding the need to have *families of models* and *families of analysis* (Davis, Bigelow, and McEver 1999). Unfortunately, government agencies often focus on a single model such as the venerable TACWAR, BRAWLER, or JANUS.

The niche of exploratory analysis is the top left hand corner of the matrix in Figure 4, which emphasizes analytical agility and *breadth* of analysis, rather than depth. However, the technique can be used hierarchically if one has a suitably modularized system model. One can do top-level exploration first and then zoom in. This is easier said than done, however, especially with traditional models. Specially designed models make things much easier, as discussed in what follows.

### 3 TECHNOLOGICAL ENABLERS

#### 3.1 The Curse of Dimensionality

In principle, exploratory analysis can be accomplished with any model. In practice, it becomes difficult with large models. If  $F$  represents the model, it can be considered to be simply a complicated function of many variables. How can we run a computerized version of  $F$  to understand its character? If  $F$  has  $M$  inputs with uncertain values, then we could consider  $N$  values for each input, construct a full factorial design (or some subset, using an experimental design and sampling), run the cases, and thereby have a characterization. However, the number of such cases would grow rapidly (as  $N^M$  for full-factorial analysis), which quickly gets out of hand even with big computers. Quite aside from setup-and-run-time issues, comprehending and communicating the consequences becomes very difficult if  $M$  is large. Suppose someone asked “Under

what conditions is  $F$  less than the danger point?” Given sufficiently powerful computers and enough time, we could create a data base of all the cases, after which we could respond to the question by spewing out lists of the cases in which  $F$  fell below the danger point. The list, however, might go on for thousands of pages. What would we do with the list? This is one manifestation of the curse of dimensionality.

#### 3.2 The Need for Abstractions

It follows that, even if we have a perfect high-resolution model, we need abstractions to use it well. And, in the dominant case in which the high-resolution model is by no means perfect, we need abstractions that allow us to ponder the phenomena in meaningful ways, with relatively small numbers of cognitive chunks. People can reason with 3, 5, or 10 such cognitive chunks at a time, but not with hundreds. If the problem is truly complex, we must find ways to organize our reasoning. That is, we must decompose the problem by using principles of modularity and hierarchy. The need for an aspect of hierarchical organization is inescapable in most systems of interest—even though the system may be highly distributed and relatively nonhierarchical in an organizational sense.

A corollary of our need for abstractions is that *we need models that use the various abstractions as inputs*. It is not sufficient merely to display the abstracts as intermediate outputs (displays) of the ultimate detailed model. The reasons include the fact that when a decision maker asks a what-if question using abstractions, there is a 1:n mapping problem in translating his question into the inputs of a more detailed model. So also when one obtains macroscopic empirical information and tries to use it for calibration. Although analysts can trick the model by selecting a mapping, doing so can be cumbersome and treacherous. It is often better if the question can be answered by a model that accepts the abstractions as inputs.

#### 3.3 Finding the Abstractions

Given the need for abstractions, how do we find them and how do we exploit them? Some guidelines are emerging (Davis and Bigelow 1998).

##### 3.3.1 When Conceiving New Models and Families

With new models, the issue is how to *design*. Several options here are as follows:

- Design the models and model families top down so that significant abstractions are built in from the start, but do so with enough understanding of

the microscopics so that the top-down design is valid.

- Design the models and families bottom up, but with enough top-down insight to assure good intermediate-level abstractions from the start.
- Do either or both of the above, but with designs taken from different perspectives.

The list does not include a pure top-down or pure bottom-up design approach. Only seldom will either generate a good design of a complex system. Note also the idea of alternative perspectives. For example, those in combat arms may conceive military problems differently than logisticians, and even more differently than historians attempting a macro-view explanation of events.

### 3.3.2 When Dealing with Existing Models

Only sometimes do we have the opportunity to design from scratch. More typically, we must adapt existing models. Moreover, the model “families” we may have to work with are often families more on the basis of assertion than lineage. What do we then do? Some possibilities here are:

- Study the model and the questions that users ask of the model to discover useful abstractions. For example, inputs  $X$ ,  $Y$ , and  $Z$  may enter the computations only as the product  $XYZ$ . Or a decision maker may ask questions in terms of concepts like force ratio. For mature models, the displays that have been added over time provide insights into useful abstractions.
- Apply statistical machinery to search for useful abstractions. For example, such machinery might test to see whether the system’s behavior correlates not just with  $X$ ,  $Y$ , and  $Z$ , but with  $XY$ ,  $XZ$ ,  $YZ$ , or  $XYZ$ .
- Idealize the system mathematically and combine this with physical insight or empirical observation to guess at the form of aggregate behavior (e.g., inverse dependence on one variable, or exponential dependence on another). Consider approximations such as an integral being the product of the effective width of the integration interval and a representative non-zero value of the integrand.

The first approach is perhaps a natural activity for a smart modeler and programmer who begins to study an existing program, but only if he open-minded about the usefulness of higher-level depictions. The second approach is an extension of normal statistical analysis. The third approach is a hybrid that I typically prefer to the second. It uses one’s understanding of phenomenology, and theories of system behavior, to gain insights about the

likely or possible abstractions *before* cranking statistical machinery.

### 3.3.3 The Problem with Occam’s Razor

The principle of Occam’s razor requires that we prefer the simplest explanation and, thus, the simplest model. Enthusiasts of statistical approaches tend to interpret this to mean that one should minimize the number of variables. They tend to focus on data and to avoid adding variables for “explanation” if the variables are not needed to predict the data. In contrast, subject-area phenomenologists may prefer to enrich the depiction by adding variables that provide a better picture of cause-effect chains, but go well beyond what can be supported with meager experimental data. My own predilection is that of the phenomenologist, but with MRM designs one can sometimes have one’s cake and eat it: one can test results empirically by focusing on the abstract versions of a model, while using richer versions for deeper explanation.

As an aside, a version of the Occam’s Razor principle emphasizes use of the explanation that is simplest enough to explain all there is to explain, but nothing simpler! This should include phenomena that one “knows about” even if they are not clearly visible in the limited data. I would add to this the admonition made decades ago by MIT’s Jay Forrester that to omit showing a variable explicitly may be equivalent to assuming its value is unity.

Competition among approaches can be useful. For example, phenomenologists working a problem may be convinced that a problem must be described with complex computer programs having hundreds or thousands of data elements. A statistical analysis may show that, despite the model’s apparent richness, the system’s resulting behavior is driven by something much simpler. This, in turn, may lead to a reconceptualizing of the problem phenomenologically. Many analogues exist in physics and engineering.

### 3.3.4 Connections Between New and Old Models

Although the discussion in Section 4.3.2 distinguished sharply between the case of new models and old ones, the reader may have noticed connections. In essence, working with existing models should often involve sketching what the models *should* be like and how models with different resolution *should* connect substantively. That is, working with existing models may require us to go back to design issues. Individuals differ, but I, at least, often find it easier to engage the problem than to engage someone’s else’s idiosyncratically described solution. Furthermore, I then have a better understanding of assumptions and approximations.

With this background, let me now turn to the design of multiresolution, multiperspective models and families

(Davis and Bigelow 1999). Although this relates most directly to new models, it is relevant also to working with legacy models in preparing for exploratory analysis.

### 3.4 Multiresolution, Multiperspective Modeling

#### 3.4.1 Definition

Multi-resolution modeling (MRM) is building a single model, a family of models, or both to describe the same phenomena at different levels of resolution, *and* to allow users to input parameters at those different levels depending on their needs. Variables at level  $n$  are abstractions of variables at level  $n+1$ . MRM is sometimes called variable-or-selectable-resolution modeling. Figure 5 illustrates MRM schematically. It indicates that a higher level model (Model A) itself has more than one level of resolution. It can be used with either two or four inputs. However, in addition to its own MRM features, it has input variables that can either be specified directly or determined from the outputs of separate higher-resolution models (models B and C, shown as “on the side,” for use when needed. In principle, one could attach models B and C in the software itself—creating a bigger model. However, in practice there are tradeoffs between doing that or keeping the more detailed models separate. For larger models and simulations, a combination single-model/family-of-models approach is desirable. This balances needs for analytical agility and complexity management.

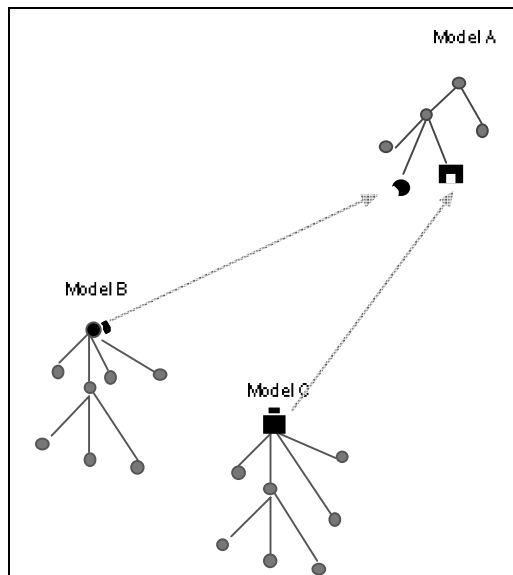


Figure 5: A Multiresolution Family

MRM is not sufficient by itself because of the need for different abstractions or perspectives in different applications. That is, different perspectives—analogue to alternative representations in physics—are legitimate and

important. They vary by conception of the system and choice of variables. Designing for both multiple resolution and multiple perspectives can be called MRMPM (pronounced Mr. MIPM).

#### 3.4.2 Mutual Calibration within a Model Family

Given MRMPM models or families, we want to be able to reconcile the concepts and predictions among levels and perspectives. It is often assumed that the correct way to do this is to calibrate upward: treating the information of the most detailed model as correct and using it to calibrate the higher-level models. This is often appropriate, but the fact is that the more detailed models almost always have omissions and shortcomings. Further, different models of a family draw upon different sources of information—ranging from doctrine or even “lore” on one extreme to physical measurements on a test range at the other.

Figure 6 makes the point that members of a multiresolution model family should be *mutually* calibrated (National Research Council 1997). For example, we may use low-resolution historical attrition or movement rates to help calibrate more detailed models predicting attrition and movement. This is not straightforward and is often done crudely by applying an overall scaling factor (fudge factor), rather than correcting the more atomic features of the detailed model, but it is likely familiar to readers. On the other hand, much calibration is indeed upward. For example, a combat model with attrition coefficients should typically have adjustments of those coefficients for different circumstances identified in a more detailed model.

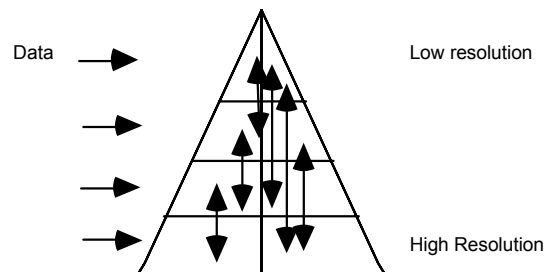


Figure 6: Mutual Calibration of Models in a Family

#### 3.4.3 Design Considerations

So, given their desirability, how do we build a family of models? Or, given pre-existing models, how do we sketch out how they “should” relate before connecting them as software or using them for mutual calibration? Some highlights are as follows.

The first design principle is to recognize that there are limits to how well lower-resolution models can be consistent with high-resolution models. *Approximation is*

a central concept from the outset. Several points are especially important:

- Consistency between two models should be assessed in the context of use. What matters is not whether they generate the same final state of the system, but whether they generate approximately the same results in the application (e.g., rank ordering of alternatives). This ties into the well-known concept of experimental frames (Zeigler, et al. 2000).
- Consistency of aggregated and disaggregated models must also be judged recognizing that low-resolution models may reflect aggregate-level knowledge not contained in the detailed model.
- Comprehensive MRM is very difficult or impossible for complex M&S, but having even some MRM can be far more useful than having none at all.
- Members of an MRM family will typically be valid for only portions of the system's state space. Parameter values (and even functional forms) should change with region.
- Mechanisms are therefore needed to recognize different situations and shift models. In simulations, human intervention is one mechanism; agent-based modeling is another.
- Valid MRM will often require stochastic variables represented by probability distributions. Further, valid aggregate models must sometimes reflect correlations among variables that might naively be seen as probabilistically independent.

With these observations, the ideal for MRM is a hierarchical design for each MRM process, as indicated in Figure 5.

### 3.4.4 Desirable Design Attributes

From the considerations we have sketched above, it follows that models and analysis methodologies for exploratory analysis should have a number of characteristics. First, they should be able to reflect hierarchical decomposition through multiple levels of resolution and from alternative perspectives representing different "aspects" of a system.

Less obviously, they should also include realistic mechanisms for the natural entities of the system to act, react, adapt, mutate, and change. These mechanisms should reflect the relative "fitness" of the original and emerging entities for the environment in which they are operating. Many techniques are applicable here, including game-theoretic methods and others that may be relatively familiar to readers. However, the most fruitful new approaches are those typically associated with the term

agent-based modeling. These include submodels that act "as the agents for" political leaders and military commanders or—at the other extreme—infantry privates on the battlefield or drivers of automobiles on the highway. In practice, such models need not be exotic: they may correspond to some relatively simple heuristic decision rules or to some well-known (though perhaps complex) operations-research algorithm. But to have such decision models is quite different from depending on scripts.

Because it is implausible that closed computer models will be able to meet the above challenge in the foreseeable future, the family of "models" should allow for human interaction—whether in human-only seminar games, small-scale model-supported human gaming, or distributed interactive simulation. This runs against the grain of much common practice.

### 3.4.5 Stochastic Inputs to Higher Level Models

The last item in the above list is often ignored in today's day-to-day work. Indeed, too often models that need to be stochastic are deterministic, with quantitatively serious consequences (Lucas 2000). Often, workers calibrate a high-level (aggregate) model using average outcomes of allegedly "representative" high-resolution scenarios. For example, a theater-level model's air model might be calibrated to results of detailed air-to-air simulation with Brawler, which treats individual engagement classes (e.g., 1 on 1, 1 on 2, ... 4 on 8). This may appear to establish the validity of the theater-level model, but in fact the calibration is treacherous. After all, what kinds of engagements occur may be a sensitive function of the sides' command and control systems, strategies, and weather. The calibrations really need to be accomplished on a highly study-specific basis.

Furthermore, the higher-level model inputs often need to be stochastic. Figure 7 illustrates the concept schematically for a simple problem. Suppose that a process (e.g., one computing the losses to aircraft in air-to-air encounters) depends on  $X, Y, S,$  and  $W$ . But suppose that the outcome of ultimate interest involves many instances of that process with different values of  $S$  and  $W$  (e.g., different per-engagement numbers of Red and Blue aircraft). An abstraction of the model might depend only on  $X, Y,$  and  $Z$  (e.g., overall attrition might depend on only numbers of Red and Blue aircraft, their relative quality, and some command and control factor). If the abstraction shown is to be valid, the variable  $Z$  should be consistent with the higher-resolution results. However, if it does not depend explicitly on  $S$  and  $W$ , then there are "hidden variables" in the problem and  $Z$  may appear to be a random variable, in which case so also would the predicted outcome  $F$  be a random variable. One could ignore this randomness if the distribution were narrow enough, but it might not be.



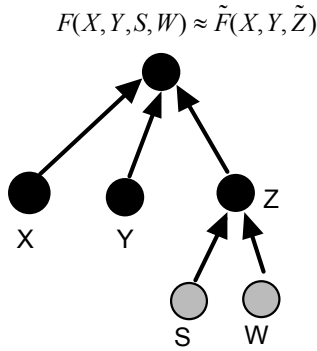


Figure 7: Input to Higher Level Model May Be Stochastic

In the past, such calibrations have been rare because analysts have lacked both theory and tools for doing things better. The “theory” part includes not having good descriptions of how the detailed model should relate to the simplified one. The tool part includes the problem of being able to define the set of runs that should be done (representing the integral of Figure 7) and then actually making those runs.

Ideally, such a calibration would be dynamic within a simulation. Moreover, it would be easy to adjust the calibration to represent different assumptions about command, control, communications, computers, intelligence, surveillance, and reconnaissance (C4ISR), as well as tactics. We are nowhere near that happy situation today,

#### 4 RECENT EXPERIENCE AND CONCLUSIONS

MRMPM is not just idealized theory, but something usable. Over the last several years, my colleagues and I have done considerable work related to the problem of halting an invading army using precision fires from aircraft and missiles. The most recent aspects of that work included understanding in some detail how the effectiveness of such fires are affected by details of terrain, enemy maneuver tactics, certain aspects of command and control, and so on. This provided a good test bed for exploring numerous aspects of MRMPM theory (Davis, Bigelow, and McEver 2000).

For this work we developed a multiresolution personal-computer model (PEM), written in Analytica, to understand and extend to other circumstances the findings from entity-level simulation of ground maneuver and long-range precision fires. A major part of that work was learning how to inform and calibrate PEM to the entity-level work. There was no possibility, in this instance, of revising the entity-level model. Nor, in practice, did we have such a good understanding of the model as to allow us to construct a comprehensive calibration theory. Instead, we had to construct a new, more abstract, model and attempt to impose

some of its abstractions on the data from runs of the entity-level simulation in prior work, plus some special runs made for our purposes. The result is a case history with what are probably some generic lessons learned.

Figure 8 illustrates one aspect of PEM’s design. It shows the data flow within a PEM module that generates the impact time (relative to the ideal impact time) for a salvo of precision weapons aimed at a packet of armored fighting vehicles observed by surveillance assets at an earlier time. Other parts of PEM combine information about packet location versus time and salvo effectiveness for targets that happen to be within the salvo’s “footprint” at the time of impact, to estimate effectiveness of precision weapons. For the salvo-impact-time module, Figure 8 shows how PEM is designed to accept inputs as detailed as whether there is enroute retargeting of weapons, the latency time, and weapon flight time. However, it can also accept more aggregate inputs such as time from last update. If the input variable Resolution of Time of Last Update Calculation is set “low,” then Time From Last Update is specified directly as input; if not, it is calculated from the lower-level inputs.

This design has proven very useful—both for analysis itself and for communicating insights to decision makers in different communities ranging from the C4ISR community to the programming and analysis community. In particular, the work clarified how the technology-intensive work of the C4ISR acquisition community relates to higher-level strategy problems and analysis of such problems at the theater level.

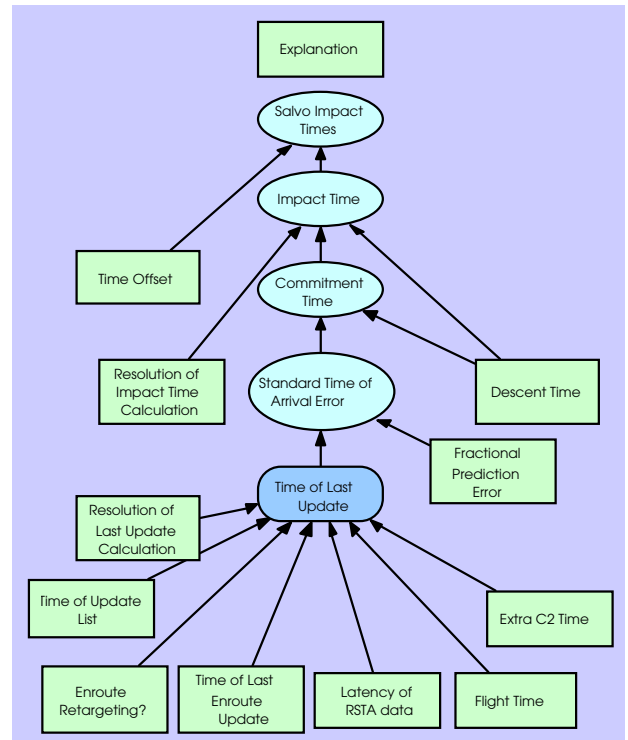


Figure 8: Multiresolution, Multiperspective Design

In other reports (McEver, Davis, and Bigelow, 2000a,b), we describe a broader but more abstract model (EXHALT) that we use for theater-level halt-problem analysis and experiments to deal with the multi-perspective problem. One conclusion is that MRMPM work rather demands a building-block approach that emphasizes study-specific assembly of the precise model needed. Although we had some success in developing a closed MRMPM model with alternative user modes representing different demands for resolution and perspective (e.g., the switches in Figure 8), it proved impossible to do very much in that regard: the number of interesting user modes and resolution combinations simply precludes being able to wire in all the relevant user modes. Moreover, that explosion of complexity occurs very quickly. At-the-time-assembly from building blocks, not prior definition, is the stronger approach. This was as we expected, but even more so.

Fortunately, we were able to construct the models needed quickly—in hours rather than days or weeks—as the result of our building-block approach, visual modeling, use of array mathematics, and strong, modular, design.

We also concluded that current personal computer tools—as powerful as they are in comparison with those in past years—are not yet up to the challenge of making the building-block/assembly approach rigorous, understandable, controllable, and reproducible without unrealistically high levels of modeler/analyst discipline. Thus, there are good challenges ahead for the enabling-technology community. Also, the search models for advanced exploratory analysis are not yet well developed.

## REFERENCES

- Bankes, S. C. 1993. Exploratory modeling for policy analysis. *Operations Research* 41(3).
- Bigelow, J. H., P. K. Davis, and J. McEver. 2000. Case history of using entity-level simulation as imperfect experimental data for informing and calibrating simpler analytical models for interdiction. In *Proceedings of the SPIE* Vol. 4026.
- Davis, P. K. (ed.). 1994. *New challenges in defense planning: Rethinking how much is enough*. RAND, Santa Monica, CA.
- Davis, P. K. 2000. Multiresolution, multiperspective modeling as an enabler of exploratory analysis. In *Proceedings of the SPIE*, Vol. 4026.
- Davis, P. K. and R. Hillestad. 2001. *Exploratory analysis for strategy problems with massive uncertainty*. RAND, Santa Monica, CA.
- Davis, P.K. and J. H. Bigelow. 1998. *Experiments in multiresolution modeling*. RAND, Santa Monica, CA.
- Davis, P.K. and M. Carrillo. 1997. *Exploratory analysis of the halt problem: A briefing on methods and initial insights*. DB-232, RAND, Santa Monica, CA.
- Davis, P. K., D. Gompert, R. Hillestad, and S. Johnson. 1998. *Transforming the Force: Suggestions for DoD Strategy*. RAND Issue Paper, Santa Monica, CA.
- Davis, P. K., D. Gompert, and R. Kugler. 1996. *Adaptiveness in national defense: the basis of a new framework*. RAND Issue Paper, Santa Monica, CA.
- Davis, P.K., J. H. Bigelow, and J. McEver. 2000. *Effects of terrain, maneuver, tactics, and C4ISR on the effectiveness of long range precision fire*. RAND, Santa Monica, CA.
- Davis, P.K., J. H. Bigelow, and J. McEver. 1999. *Analytic methods for studies and experiments on transforming the Force*. RAND, Santa Monica, CA.
- Lempert, R., M. E. Schlesinger, and S. C. Bankes. 1996. When we don't know the costs or the benefits: adaptive strategies for abating climate change. *Climatic Change* 33(2).
- Lucas, Thomas. 2000. The stochastic versus deterministic argument for combat simulations: tales of when the average won't do. *Military Operations Research* 5(3).
- McEver, J., P. K. Davis, and J. H. Bigelow. 2000a. *EXHALT: An interdiction model for the halt phase of armored invasions*. RAND, Santa Monica, CA.
- McEver, J., P. K. Davis, and J.H. Bigelow. 2000b. Implementing multiresolution models and families of models: from entity level simulation to personal-computer stochastic models and simple 'repro models.' *Proceedings of the SPIE* Vol. 4026.
- Morgan, G., and M. Henrion. 1992. *Uncertainty: A guide to dealing with uncertainty in quantitative risk and policy analysis*. Cambridge, MA: Cambridge University Press.
- National Research Council. 1997. *Modeling and simulation: Technology for the United States Navy and Marine Corps, 2000–2035* Vol. 9. Washington, D.C.: National Academy Press.
- Zeigler, B. P., T. G. Kim, and H. Praehofer. 2000. *Theory of modeling and design*. San Diego: Academic Press.

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