Government of Western Australia
School Curriculum and Standards Authority

MATHEMATICS METHODS

ATAR COURSE

Year 11 syllabus

IMPORTANT INFORMATION

This syllabus is effective from 1 January 2015.

Users of this syllabus are responsible for checking its currency.

Syllabuses are formally reviewed by the School Curriculum and Standards Authority on a cyclical basis, typically every five years.

Content

Overview of mathematics courses

There are six mathematics courses, three General and three ATAR. Each course is organised into four units. Unit 1 and Unit 2 are taken in Year 11 and Unit 3 and Unit 4 in Year 12. The ATAR course examination for each of the three ATAR courses is based on Unit 3 and Unit 4 only.

The courses are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Mathematics Preliminary is a General course which focuses on the practical application of knowledge, skills and understandings to a range of environments that will be accessed by students with special education needs. Grades are not assigned for these units. Student achievement is recorded as ‘completed’ or ‘not completed’. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Foundation is a General course which focuses on building the capacity, confidence and disposition to use mathematics to meet the numeracy standard for the Western Australian Certificate of Education (WACE). It provides students with the knowledge, skills and understanding to solve problems across a range of contexts, including personal, community and workplace/employment. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Essential is a General course which focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Applications is an ATAR course which focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering questions that involve analysing univariate and bivariate data, including time series data.

Mathematics Methods is an ATAR course which focuses on the use of calculus and statistical analysis. The study of calculus provides a basis for understanding rates of change in the physical world, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics develops students’ ability to describe and analyse phenomena that involve uncertainty and variation.

Mathematics Specialist is an ATAR course which provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. The Mathematics Specialist ATAR course contains topics in functions and calculus that build on and deepen the ideas presented in the Mathematics Methods ATAR course, as well as demonstrate their application in many areas. This course also extends understanding and knowledge of statistics and introduces the topics of vectors, complex numbers and matrices. The Mathematics Specialist ATAR course is the only ATAR mathematics course that should not be taken as a stand-alone course.

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring, it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics are concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real-world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

The major themes of the Mathematics Methods ATAR course are calculus and statistics. They include, as necessary prerequisites, studies of algebra, functions and their graphs, and probability. They are developed systematically, with increasing levels of sophistication and complexity. Calculus is essential for developing an understanding of the physical world because many of the laws of science are relationships involving rates of change. Statistics is used to describe and analyse phenomena involving uncertainty and variation. For these reasons, this course provides a foundation for further studies in disciplines in which mathematics and statistics have important roles. It is also advantageous for further studies in the health and social sciences. This course is designed for students whose future pathways may involve mathematics and statistics and their applications in a range of disciplines at the tertiary level.

For all content areas of the Mathematics Methods course, the proficiency strands of the Year 7–10 curriculum continue to be applicable and should be inherent in students' learning of this course. These strands are Understanding, Fluency, Problem-solving and Reasoning, and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency in skills, such as calculating derivatives and integrals, or solving quadratic equations, and frees up working memory for more complex aspects of problem solving. The ability to transfer skills to solve problems based on a wide range of applications is a vital part of this course. Because both calculus and statistics are widely applicable as models of the world around us, there is ample opportunity for problem-solving throughout the course.

The Mathematics Methods ATAR course is structured over four units. The topics in Unit 1 build on students' mathematical experience. The topics 'Functions and graphs', 'Trigonometric functions' and 'Counting and probability' all follow on from topics in the Year 7–10 curriculum from the strands Number and Algebra, Measurement and Geometry, and Statistics and Probability. In this course, there is a progression of content and applications in all areas. For example, in Unit 2 differential calculus is introduced, and then further developed in Unit 3, where integral calculus is introduced. Discrete probability distributions are introduced in Unit 3, and then continuous probability distributions and an introduction to statistical inference conclude Unit 4.

Aims

The Mathematics Methods ATAR course aims to develop students':

- understanding of concepts and techniques drawn from algebra, the study of functions, calculus, probability and statistics
- ability to solve applied problems using concepts and techniques drawn from algebra, functions, calculus, probability and statistics
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information, including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately and efficiently.

Organisation

This course is organised into a Year 11 syllabus and a Year 12 syllabus. The cognitive complexity of the syllabus content increases from Year 11 to Year 12.

Structure of the syllabus

The Year 11 syllabus is divided into two units, each of one semester duration, which are typically delivered as a pair. The notional time for each unit is 55 class contact hours.

Organisation of content

Unit 1

Contains the three topics:

- Functions and graphs
- Trigonometric functions
- Counting and probability.

Unit 1 begins with a review of the basic algebraic concepts and techniques required for a successful introduction to the study of functions and calculus. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of probability and statistics begins in this unit with a review of the fundamentals of probability, and the introduction of the concepts of conditional probability and independence. The study of the trigonometric functions begins with a consideration of the unit circle using degrees and the trigonometry of triangles and its application. Radian measure is introduced, and the graphs of the trigonometric functions are examined and their applications in a wide range of settings are explored.

Unit 2

Contains the three topics:

- Exponential functions
- Arithmetic and geometric sequences and series
- Introduction to differential calculus.

In Unit 2, exponential functions are introduced and their properties and graphs examined. Arithmetic and geometric sequences and their applications are introduced and their recursive definitions applied. Rates and average rates of change are introduced and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically (by calculating difference quotients), geometrically (as slopes of chords and tangents), and algebraically. This first calculus topic concludes with derivatives of polynomial functions, using simple applications of the derivative to sketch curves, calculate slopes and equations of tangents, determine instantaneous velocities, and solve optimisation problems.

Each unit includes:

- a unit description – a short description of the focus of the unit
- learning outcomes – a set of statements describing the learning expected as a result of studying the unit
- unit content – the content to be taught and learned.

Role of technology

It is assumed that students will be taught this course with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of the course. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in this course.

Progression from the Year 7–10 curriculum

In this syllabus, there is a strong emphasis on mutually reinforcing proficiencies in Understanding, Fluency, Problem-solving and Reasoning. Students gain fluency in a variety of mathematical and statistical skills, including algebraic manipulations, constructing and interpreting graphs, calculating derivatives and integrals, applying probabilistic models, estimating probabilities and parameters from data, and using appropriate technologies. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem-solving. In order to study this course, it is desirable that students complete topics from 10A. The knowledge and skills from the following content descriptions from 10A are highly recommended for the study of the Mathematics Methods ATAR course.

- ACMNA264: Define rational and irrational numbers, and perform operations with surds and fractional indices
- ACMNA269: Factorise monic and non-monic quadratic expressions, and solve a wide range of quadratic equations derived from a variety of contexts
- ACMSP278: Calculate and interpret the mean and standard deviation of data, and use these to compare datasets

Representation of the general capabilities

The general capabilities encompass the knowledge, skills, behaviours and dispositions that will assist students to live and work successfully in the twenty-first century. Teachers may find opportunities to incorporate the capabilities into the teaching and learning program for the Mathematics Methods ATAR course. The general capabilities are not assessed unless they are identified within the specified unit content.

Literacy

Literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy

Students who undertake this course will continue to develop their numeracy skills at a more sophisticated level, making decisions about the relevant mathematics to use, following through with calculations selecting appropriate methods and being confident of their results. This course contains topics that will equip students for the ever-increasing demands of the information age, developing the skills of critical evaluation of numerical information. Students will enhance their numerical operation skills via engagement with number sequences, trigonometric and probability calculations, algebraic relationships and the development of calculus concepts.

Information and communication technology capability

Students use information and communication technology (ICT) both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved, such as for statistical analysis, generation of algorithms, manipulation and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and creative thinking

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and social capability

Students develop personal and social competence in mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision making. The elements of personal and social competence relevant to mathematics mainly include the application of mathematical skills for their decision making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical understanding

Students develop ethical understanding in mathematics through decision making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results, and the social responsibility associated with teamwork and attribution of input.

The areas relevant to mathematics include issues associated with ethical decision making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and ethical understanding. Students develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding

Students understand mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person’s environment and language.

Representation of the cross-curriculum priorities

The cross-curriculum priorities address contemporary issues which students face in a globalised world. Teachers may find opportunities to incorporate the priorities into the teaching and learning program for the Mathematics Methods ATAR course. The cross-curriculum priorities are not assessed unless they are identified within the specified unit content.

Aboriginal and Torres Strait Islander histories and cultures

Mathematics courses value the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples’ past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students’ understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples’ histories and cultures.

Asia and Australia's engagement with Asia

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia’s place in the region. By analysing relevant data, students have opportunities to further develop an understanding of the diverse nature of Asia’s environments and traditional and contemporary cultures.

Sustainability

Each of the mathematics courses provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Teachers are therefore encouraged to select contexts for discussion that are connected with sustainability. Through the analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

Unit 1

Unit description

This unit begins with a review of the basic algebraic concepts and techniques required for a successful introduction to the study of calculus. The basic trigonometric functions are then introduced. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of inferential statistics begins in this unit with a review of the fundamentals of probability and the introduction of the concepts of counting, conditional probability and independence. Access to technology to support the computational and graphical aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in algebra, functions, graphs, trigonometric functions, counting and probability
- solve problems using algebra, functions, graphs, trigonometric functions, counting and probability
- apply reasoning skills in the context of algebra, functions, graphs, trigonometric functions, counting and probability
- interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems
- communicate their arguments and strategies when solving problems.

Unit content

This unit includes the knowledge, understandings and skills described below.

Topic 1.1: Functions and graphs (22 hours)

Lines and linear relationships

1.1.1 determine the coordinates of the mid-point between two points

1.1.2 determine an end-point given the other end-point and the mid-point

1.1.3 examine examples of direct proportion and linearly related variables

1.1.4 recognise features of the graph of $y = mx + c$, including its linear nature, its intercepts and its slope or gradient

1.1.5 determine the equation of a straight line given sufficient information; including parallel and perpendicular lines

1.1.6 solve linear equations, including those with algebraic fractions and variables on both sides

Quadratic relationships

1.1.7 examine examples of quadratically related variables

1.1.8 recognise features of the graphs of $y = x^2$, $y = a(x - b)^2 + c$, and $y = a(x - b)(x - c)$, including their parabolic nature, turning points, axes of symmetry and intercepts

1.1.9 solve quadratic equations, including the use of quadratic formula and completing the square

1.1.10 determine the equation of a quadratic given sufficient information

1.1.11 determine turning points and zeros of quadratics and understand the role of the discriminant

1.1.12 recognise features of the graph of the general quadratic $y = ax^2 + bx + c$

Inverse proportion

1.1.13 examine examples of inverse proportion

1.1.14 recognise features and determine equations of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x-b}$, including their hyperbolic shapes and their asymptotes.

Powers and polynomials

1.1.15 recognise features of the graphs of $y = x^n$ for $n \in \boldsymbol{N}, n = -1$ and $n = ½$, including shape, and behaviour as $x \to \infty$ and $x \to -\infty$

1.1.16 identify the coefficients and the degree of a polynomial

1.1.17 expand quadratic and cubic polynomials from factors

1.1.18 recognise features and determine equations of the graphs of $y = x^3$, $y = a(x - b)^3 + c$ and $y = k(x - a)(x - b)(x - c)$, including shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$

1.1.19 factorise cubic polynomials in cases where a linear factor is easily obtained

1.1.20 solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained

Graphs of relations

1.1.21 recognise features and determine equations of the graphs of $x^2 + y^2 = r^2$ and $(x - a)^2 + (y - b)^2 = r^2$, including their circular shapes, their centres and their radii

1.1.22 recognise features of the graph of $y^2 = x$, including its parabolic shape and its axis of symmetry

Functions

1.1.23 understand the concept of a function as a mapping between sets and as a rule or a formula that defines one variable quantity in terms of another

1.1.24 use function notation; determine domain and range; recognise independent and dependent variables

1.1.25 understand the concept of the graph of a function

1.1.26 examine translations and the graphs of $y = f(x) + a$ and $y = f(x - b)$

1.1.27 examine dilations and the graphs of $y = cf(x)$ and $y = f(dx)$

1.1.28 recognise the distinction between functions and relations and apply the vertical line test

Topic 1.2: Trigonometric functions (15 hours)

Cosine and sine rules

1.2.1 review sine, cosine and tangent as ratios of side lengths in right-angled triangles

1.2.2 understand the unit circle definition of $\cos\theta$, $\sin\theta$ and $\tan\theta$ and periodicity using degrees

1.2.3 examine the relationship between the angle of inclination of a line and the gradient of that line

1.2.4 establish and use the cosine and sine rules, including consideration of the ambiguous case and the formula $Area = \frac{1}{2}bc\ sin\ A$ for the area of a triangle

Circular measure and radian measure

1.2.5 define and use radian measure and understand its relationship with degree measure

1.2.6 calculate lengths of arcs and areas of sectors and segments in circles

Trigonometric functions

1.2.7 understand the unit circle definition of $\sin\theta$, $\cos\theta$ and $\tan\theta$ and periodicity using radians

1.2.8 recognise the exact values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$

1.2.9 recognise the graphs of $y = \sin x, y = \cos x$, and $y = \tan x$ on extended domains

1.2.10 examine amplitude changes and the graphs of $y = a\sin x$ and $y = a\cos x$

1.2.11 examine period changes and the graphs of $y = \sin bx$, $y = \cos bx$ and $y = \tan\ bx$

1.2.12 examine phase changes and the graphs of $y = \sin(x - c)$, $y = \cos(x - c)$ and $y = \tan\ (x - c)$

1.2.13 examine the relationships $\sin\left(x + \frac{\pi}{2}\right) = \cos x$ and $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

1.2.14 prove and apply the angle sum and difference identities

1.2.15 identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems

1.2.16 solve equations involving trigonometric functions using technology, and algebraically in simple cases

Topic 1.3: Counting and probability (18 hours)

Combinations

1.3.1 understand the notion of a combination as a set of r objects taken from a set of n distinct objects

1.3.2 use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of r objects taken from a set of n distinct objects

1.3.3 expand $(x + y)^n$ for small positive integers n

1.3.4 recognise the numbers $\binom{n}{r}$ as binomial coefficients (as coefficients in the expansion of $(x + y)^n$)

1.3.5 use Pascal's triangle and its properties

Language of events and sets

1.3.6 review the concepts and language of outcomes, sample spaces, and events, as sets of outcomes

1.3.7 use set language and notation for events, including:

a. $\bar{A}$ (or A') for the complement of an event A

b. $A \cap B$ and $A \cup B$ for the intersection and union of events A and B respectively

c. $A \cap B \cap C$ and $A \cup B \cup C$ for the intersection and union of the three events A, B and C respectively

d. recognise mutually exclusive events.

1.3.8 use everyday occurrences to illustrate set descriptions and representations of events and set operations

Review of the fundamentals of probability

1.3.9 review probability as a measure of 'the likelihood of occurrence' of an event

1.3.10 review the probability scale: $0 \leq P(A) \leq 1$ for each event A, with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certainty

1.3.11 review the rules: $P(\overline{\mathrm{A}}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

1.3.12 use relative frequencies obtained from data as estimates of probabilities

Conditional probability and independence

1.3.13 understand the notion of a conditional probability and recognise and use language that indicates conditionality

1.3.14 use the notation $P(A|B)$ and the formula $P(A \cap B) = P(A|B)P(B)$

1.3.15 understand the notion of independence of an event A from an event B, as defined by $P(A|B) = P(A)$

1.3.16 establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B, and recognise the symmetry of independence

1.3.17 use relative frequencies obtained from data as estimates of conditional probabilities and as indications of possible independence of events

Unit 2

Unit description

The algebra section of this unit focuses on exponentials. Their graphs are examined and their applications in a wide range of settings are explored. Arithmetic and geometric sequences are introduced and their applications are studied. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically, by calculating difference quotients both geometrically as slopes of chords and tangents, and algebraically. Calculus is developed to study the derivatives of polynomial functions, with simple application of the derivative to curve sketching, the calculation of slopes and equations of tangents, the determination of instantaneous velocities and the solution of optimisation problems. The unit concludes with a brief consideration of anti-differentiation.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques used in algebra, sequences and series, functions, graphs and calculus
- solve problems in algebra, sequences and series, functions, graphs and calculus
- apply reasoning skills in algebra, sequences and series, functions, graphs and calculus
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems
- communicate arguments and strategies when solving problems.

Unit content

This unit builds on the content covered in Unit 1.

This unit includes the knowledge, understandings and skills described below.

Topic 2.1: Exponential functions (10 hours)

Indices and the index laws

2.1.1 review indices (including fractional and negative indices) and the index laws

2.1.2 use radicals and convert to and from fractional indices

2.1.3 understand and use scientific notation and significant figures

Exponential functions

2.1.4 establish and use the algebraic properties of exponential functions

2.1.5 recognise the qualitative features of the graph of $y = a^x$ $(a > 0)$, including asymptotes, and of its translations ($y = a^x + b$ and $y = a^{x-c}$)

2.1.6 identify contexts suitable for modelling by exponential functions and use them to solve practical problems

2.1.7 solve equations involving exponential functions using technology, and algebraically in simple cases

Topic 2.2: Arithmetic and geometric sequences and series (15 hours)

Arithmetic sequences

2.2.1 recognise and use the recursive definition of an arithmetic sequence: $t_{n+1} = t_n + d$

2.2.2 develop and use the formula $t_n = t_1 + (n-1)d$ for the general term of an arithmetic sequence and recognise its linear nature

2.2.3 use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest

2.2.4 establish and use the formula for the sum of the first n terms of an arithmetic sequence

Geometric sequences

2.2.5 recognise and use the recursive definition of a geometric sequence: $t_{n+1} = t_n r$

2.2.6 develop and use the formula $t_n = t_1 r^{n-1}$ for the general term of a geometric sequence and recognise its exponential nature

2.2.7 understand the limiting behaviour as $n \to \infty$ of the terms t_n in a geometric sequence and its dependence on the value of the common ratio r

2.2.8 establish and use the formula $S_n = t_1 \frac{r^n - 1}{r - 1}$ for the sum of the first n terms of a geometric sequence

2.2.9 use geometric sequences in contexts involving geometric growth or decay, such as compound interest

Topic 2.3: Introduction to differential calculus (30 hours)

Rates of change

2.3.1 interpret the difference quotient $\frac{f(x+h) - f(x)}{h}$ as the average rate of change of a function f

2.3.2 use the Leibniz notation δx and δy for changes or increments in the variables x and y

2.3.3 use the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h) - f(x)}{h}$ where $y = f(x)$

2.3.4 interpret the ratios $\frac{f(x+h) - f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y = f(x)$

The concept of the derivative

2.3.5 examine the behaviour of the difference quotient $\frac{f(x+h) - f(x)}{h}$ as $h \to 0$ as an informal introduction to the concept of a limit

2.3.6 define the derivative $f'(x)$ as $\lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

2.3.7 use the Leibniz notation for the derivative: $\frac{dy}{dx} = \lim\limits_{\delta x \to 0} \frac{\delta y}{\delta x}$ and the correspondence $\frac{dy}{dx} = f'(x)$ where $y = f(x)$

2.3.8 interpret the derivative as the instantaneous rate of change

2.3.9 interpret the derivative as the slope or gradient of a tangent line of the graph of $y = f(x)$

Computation of derivatives

2.3.10 estimate numerically the value of a derivative for simple power functions

2.3.11 examine examples of variable rates of change of non-linear functions

2.3.12 establish the formula $\frac{d}{dx}(x^n) = nx^{n-1}$ for non-negative integers n expanding $(x+h)^n$ or by factorising $(x+h)^n - x^n$

Properties of derivatives

2.3.13 understand the concept of the derivative as a function

2.3.14 identify and use linearity properties of the derivative

2.3.15 calculate derivatives of polynomials

Applications of derivatives

2.3.16 determine instantaneous rates of change

2.3.17 determine the slope of a tangent and the equation of the tangent

2.3.18 construct and interpret position-time graphs with velocity as the slope of the tangent

2.3.19 recognise velocity as the first derivative of displacement with respect to time

2.3.20 sketch curves associated with simple polynomials, determine stationary points, and local and global maxima and minima, and examine behaviour as $x \to \infty$ and $x \to -\infty$

2.3.21 solve optimisation problems arising in a variety of contexts involving polynomials on finite interval domains

Anti-derivatives

2.3.22 calculate anti-derivatives of polynomial functions

School-based assessment

The Western Australian Certificate of Education (WACE) Manual contains essential information on principles, policies and procedures for school-based assessment that needs to be read in conjunction with this syllabus.

Teachers design school-based assessment tasks to meet the needs of students. The table below provides details of the assessment types for the Mathematics Methods ATAR Year 11 syllabus and the weighting for each assessment type.

Assessment table – Year 11

Type of assessment	Weighting
Response Students respond using knowledge of mathematical facts, concepts and terminology, applying problem-solving skills and. Response tasks can include: tests, assignments, quizzes and observation checklists. Tests are administered under controlled and timed conditions.	40%
Investigation Students plan, research, conduct and communicate the findings of an investigation. They can investigate problems to identify the underlying mathematics, or select, adapt and apply models and procedures to solve problems. This assessment type provides for the assessment of general inquiry skills, course-related knowledge and skills, and modelling skills. Evidence can include: observation and interview, written work or multimedia presentations.	20%
Examination Students apply mathematical understanding and skills to analyse, interpret and respond to questions and situations. Examinations provide for the assessment of conceptual understandings, knowledge of mathematical facts and terminology, problem-solving skills, and the use of algorithms. Examination questions can range from those of a routine nature, assessing lower level concepts, through to open-ended questions that require responses at the highest level of conceptual thinking. Students can be asked questions of an investigative nature for which they may need to communicate findings, generalise, or make and test conjectures. Typically conducted at the end of each semester and/or unit. In preparation for Unit 3 and Unit 4, the examination should reflect the examination design brief included in the ATAR Year 12 syllabus for this course. Where a combined assessment outline is implemented, the Semester 2 examination should assess content from both Unit 1 and Unit 2. However, the combined weighting of Semester 1 and Semester 2 should reflect the respective weightings of the course content as a whole.	40%

Teachers are required to use the assessment table to develop an assessment outline for the pair of units (or for a single unit where only one is being studied).

The assessment outline must:

- include a set of assessment tasks
- include a general description of each task
- indicate the unit content to be assessed
- indicate a weighting for each task and each assessment type
- include the approximate timing of each task (for example, the week the task is conducted, or the issue and submission dates for an extended task).

In the assessment outline for the pair of units

- each assessment type must be included at least twice
- the response type must include a minimum of two tests.

In the assessment outline where a single unit is being studied:

- each assessment type must be included at least once
- the response type must include at least one test.

The set of assessment tasks must provide a representative sampling of the content for Unit 1 and Unit 2.

Assessment tasks not administered under test/controlled conditions require appropriate validation/authentication processes. This may include observation, annotated notes, checklists, interview, presentations or in-class tasks assessing related content and processes.

Grading

Schools report student achievement in terms of the following grades:

Grade	Interpretation
A	Excellent achievement
B	High achievement
C	Satisfactory achievement
D	Limited achievement
E	Very low achievement

The teacher prepares a ranked list and assigns the student a grade for the pair of units (or for a unit where only one unit is being studied). The grade is based on the student's overall performance as judged by reference to a set of pre-determined standards. These standards are defined by grade descriptions and annotated work samples. The grade descriptions for the Mathematics Methods ATAR Year 11 syllabus are provided in Appendix 1. They can also be accessed, together with annotated work samples, through the Guide to Grades link on the course page of the Authority website at www.scsa.wa.edu.au

To be assigned a grade, a student must have had the opportunity to complete the education program, including the assessment program (unless the school accepts that there are exceptional and justifiable circumstances).

Refer to the WACE Manual for further information about the use of a ranked list in the process of assigning grades.

Appendix 1 – Grade descriptions Year 11

A	**Identifies and organises relevant information** Identifies and organises relevant information that is dense and scattered, for example moving from function notation to algebraic equations and solving to identify a series of features of the graph of the polynomial. Labels diagrams clearly from information given to solve bearing problems using trigonometry and works fluently with surds when required. Organises information from a given problem using Venn diagrams and follows through to calculate relevant probability values. Identifies the natural domain and range from the equation of a function.
	Chooses effective models and methods and carries the methods through correctly Carries an extended response through, for example solves complex problems involving counting and probability, correctly identifies mutually exclusive events and the number of choices that apply for each event, and applies the addition principle to arrive at a correct result. Chooses an appropriate model, such as using radians when dealing with circle measure or recognising exact trigonometric ratios in specified situations. Chooses the appropriate linear/geometric growth model to solve practical problems in context.
	Follows mathematical conventions and attends to accuracy Uses inequality signs correctly and attends to open or closed intervals when specifying function range or domain. Rounds, unprompted, to suit contexts and specified accuracies in extended responses. Follows good practice when graphing or sketching functions, attending to all the labelling and naming conventions, including identifying the key features, such as asymptotes; uses a tree diagram or multiplication principle to calculate a compound probability. Works with surds accurately when asked to give exact values.
	Links mathematical results to data and contexts to reach reasonable conclusions Uses the correct units when dealing with multidimensional formulae such as $PV=kT$. Uses counting techniques effectively to calculate probabilities where large numbers of choices are present. Draws accurate diagrams when representing transformations of functions given as graphs or in function notation. Links two or more content areas of the syllabus, such as distance travelled and geometric sequences, to solve a problem in context. Evaluates the trigonometric ratios such as $\cos\frac{\pi}{3}=\frac{\sqrt{3}}{2}$ without converting to degrees.
	Communicates mathematical reasoning, results and conclusions Shows main steps in reasoning, and sets out proofs of identities using the left hand side equals the right hand side and modifies the right hand side in steps to work towards the conclusion. Defines sides and angles in geometric diagrams when using measurement formulae such as $\text{Area}=\frac{1}{2}ab\sin\theta$. Labels Venn diagrams effectively and can correctly identify subsets such as $(A\cup B)\cap C$. Draws effective diagrams for bearings problems and uses appropriate lengths and angles with the correct formulae. Clearly sets out the working when using the quadratic formula or completing the square to solve quadratic equations that are not easily factorised and exact solutions are required.

B	**Identifies and organises relevant information** Identifies and organises relevant information, for example moving from function notation to algebraic equations. Identifies features of the graph of a polynomial such as turning points. Labels trigonometric diagrams clearly from information given and works fluently with surds. Organises information using Venn diagrams and follows through to calculate relevant probability values. Identifies the natural domain from the equation of a function.
	Chooses effective models and methods and carries the methods through correctly Carries through an extended response, for example solves geometric problems involving compound diagrams, correctly factorises cubic polynomials to solve equations. Uses the unit circle and Pythagorean triangles effectively to determine values, such as $\text{Cos}\left(\frac{5\pi}{3}\right)$. Connects two formulae in a problem to solve for a given variable, such as using the area formula and the arc length formula to solve for arc length.
	Follows mathematical conventions and attends to accuracy Rounds to the correct number of decimal places to suit contexts and specified accuracies in extended responses. Rationalises expressions with surds when required. Works effectively with equations containing indices. Uses Pascal's triangle to expand a binomial product and accurately gathers the terms to give the polynomial expression. Uses exact values as required when dealing with radian measure.
	Links mathematical results to data and contexts to reach reasonable conclusions Links the domain of a function to its range, accurately defining the range using function notation and taking note of open or closed intervals. Constructs a revenue function from a table of values having three columns, such as number of units sold, price per unit and variation in price per unit. Models the information given on a tidal change to a trigonometric function and solves related problems to the required degree of accuracy. Uses the quadratic formula to solve when the quadratic term does not factorise.
	Communicates mathematical reasoning, results and conclusions Justifies working by stating properties/conditions that have been applied, for example explaining the use of the sine/cosine rule to determine a specified length or area on a two-dimensional shape, given appropriate conditions. Shows the steps in determining the turning points of a cubic function and using these to help sketch its graph.

C	**Identifies and organises relevant information** Identifies and organises relevant information that is provided in a straight forward manner, for example x- and y-intercepts of the graph of linear functions, reads the amplitude or period off the graph of a trigonometric function; locates the line of symmetry, roots and y-intercepts of the graph of a quadratic function. Locates the centre and radius of a circle given the equation of its relation. Uses the initial value of an exponential equation of the form $V = V_0 \times a^t$ to calculate V_0. Identifies the correct ratio of a geometric sequence from a simple dialogue. Identifies the asymptotes of a simple rational function of the form $y = \frac{a}{x-b}$.
	Chooses effective models and methods and carries the methods through correctly Uses differentiation to locate the turning point and hence the maximum value of a revenue function. Uses Pythagoras's theorem to evaluate trigonometric ratios such as, $\sin A$ given $\cos A = \frac{a}{b}$ and $0^0 \le A \le 90^0$ Uses the first derivative of a function to determine the gradient of a tangent to a cubic graph at a given point. Uses the sine rule or cosine rule where appropriate.
	Follows mathematical conventions and attends to accuracy Completes the missing probabilities in a partially constructed tree diagram. Assigns probability values to simple subsets such as $P(A \cup B)$ in a universal set containing sets A and B. Completes routine calculations to evaluate the selection of *r* objects from a set of *n* distinct objects. Uses correct conventions when differentiating with function and Leibnitz notation.

	Links mathematical results to data and contexts to reach reasonable conclusions Is consistent with units when moving from displacement time graphs to velocity time graphs. Links the maximum or minimum turning point of a displacement time graph to zero velocity. Uses simultaneous equations to locate the intersection of two linear graphs. Equates the gradients of a pair of parallel linear graphs that is $m_1 = m_2$ and also that $m_1 \times m_2 = -1$ when the graphs are perpendicular.
	Communicates mathematical reasoning, results and conclusions Shows working of linear equations by using the equality sign correctly. Sets out rational expressions clearly and indicates cancelling of common factors. Shows adequate working when substituting into expressions in function notation. Interchanges $\frac{dx}{dt}$ for $v(t)$ in displacement time problems and uses appropriate units.

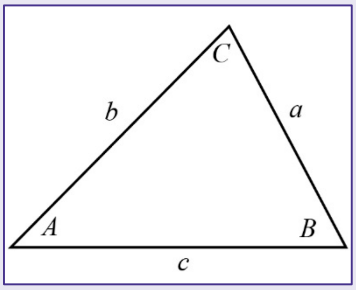
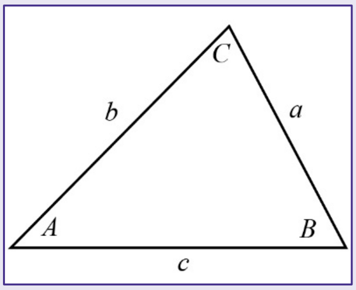
D	**Identifies and organises relevant information** Reads the coordinates of x- and y-intercepts of linear graphs. Substitutes values into given polynomial functions and usually simplifies correctly. Labels parts of geometric diagrams from given information. Uses variables on a simple diagram, for example of a rectangle to evaluate the $area = length \times width = (2x+3)(x)$ with some errors and omissions.
	Chooses effective models and methods and carries the methods through correctly Differentiates simple expressions correctly and simplifies terms when required. Uses the derivative of a function to locate a turning point. Calculates with trigonometric ratios in simple or straight forward diagrams. Uses the null factor theorem to solve simple factorised equations. Applies the circular measure formulae such as Area $= \frac{1}{2}r^2\theta$.
	Follows mathematical conventions and attends to accuracy Uses function notation and gives the coordinates of related points correctly. Uses the constant of integration with anti-differentiation. Differentiates individual terms of a polynomial function correctly. Calculates unknown angle values in simple geometric diagrams such as a cyclic quadrilateral. Factorises simple quadratic terms only, to calculate roots.
	Links mathematical results to data and contexts to reach reasonable conclusions Uses units in calculations when prompted. Recognises the implicit form of the equation of the circle. Uses the gradient intercept form $y = mx + c$ to plot linear graphs. Changes radian measure of angles to degrees such as $\frac{\pi}{3} = 60^0$ when it is not necessary. Calculates the gradient of a secant using the formula for gradient $\frac{f(b)-f(a)}{b-a}$ where $(a, f(a))$ and $(b, f(b))$ are points on the function.
	Communicates mathematical reasoning, results and conclusions Sets out substitution into simple formula and shows related calculation. Uses technology without using appropriate level of accuracy. Labels geometric diagrams with some errors. Works with coordinates to calculate gradient or mid-points between two points.

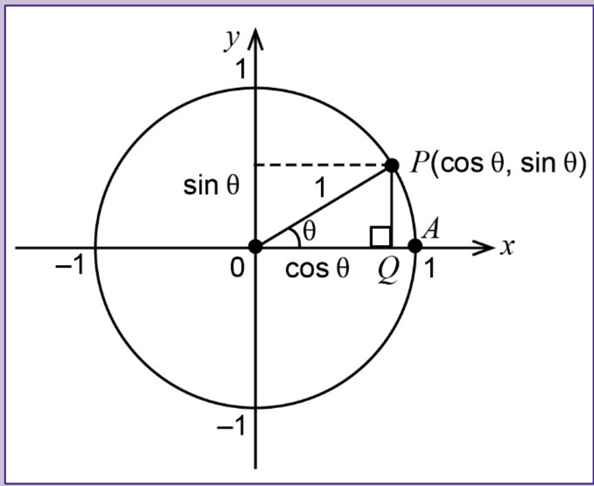
E	Does not meet the requirements of a D grade and/or has completed insufficient assessment tasks to be assigned a higher grade.

Appendix 2 – Glossary

This glossary is provided to enable a common understanding of the key terms in this syllabus.

Unit 1	
Functions and graphs	
Asymptote	A line is an asymptote to a curve if the distance between the line and the curve approaches zero as they 'tend to infinity'. For example, the line with equation $x = \frac{\pi}{2}$ is a vertical asymptote to the graph of $y = \tan x$, and the line with equation $y = 0$ is a horizontal asymptote to the graph of $y = \frac{1}{x}$.
Binomial distribution	The expansion $(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{r}x^{n-r}y^r + \cdots + y^n$ is known as the binomial theorem. The numbers $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n\times(n-1)\times\cdots\times(n-r+1)}{r\times(r-1)\times\cdots\times2\times1}$ are called binomial coefficients.
Completing the square	The quadratic expression $ax^2 + bx + c$ can be rewritten as $a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$. Re-writing it in this way is called completing the square.
Discriminant	The discriminant of the quadratic expression $ax^2 + bx + c$ is the quantity $b^2 - 4ac$.
Function	A function f is a rule that associates with each element x in a set S, a unique element $f(x)$ in a set T. We write $x \mapsto f(x)$ to indicate the mapping of x to $f(x)$. The set S is called the domain of f and the set T is called the codomain. The subset of T consisting of all the elements $f(x): x \in S$ is called the range of f. If we write $y = f(x)$ we say that x is the independent variable and y is the dependent variable.
Graph of a function	The graph of a function f is the set of all points (x, y) in Cartesian plane where x is in the domain of f and $y = f(x)$.
Quadratic formula	If $ax^2 + bx + c = 0$ with $a \neq 0$, then $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$. This formula for the roots is called the quadratic formula.
Vertical line test	A relation between two real variables x and y is a function and $y = f(x)$ for some function f, if and only if each vertical line, i.e. each line parallel to the y-axis, intersects the graph of the relation in, at most, one point. This test to determine whether a relation is, in fact, a function is known as the vertical line test.
Trigonometric functions	
Angle sum and difference identites	The angle sum and difference identites for sine and cosine are given by $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
Area of a sector	The area of a sector of a circle is given by $A = \frac{1}{2}r^2\theta$, where A is the sector area, r is the radius and θ is the angle subtended at the centre, measured in radians.

Area of a segment	The area of a segment of a circle is given by $A = \frac{1}{2}r^2(\theta - \sin\theta)$, where A is the segment area, r is the radius and θ is the angle subtended at the centre, measured in radians.
Circular measure	Circular measure is the measurement of angle size in radians.
Length of an arc	The length of an arc in a circle is given by $\ell = r\theta$, where ℓ is the arc length, r is the radius and θ is the angle subtended at the centre, measured in radians. This is simply a rearrangement of the formula defining the radian measure of an angle.
Length of a chord	The length of a chord in a circle is given by $\ell = 2r\sin\frac{1}{2}\theta$, where ℓ is the chord length, r is the radius and θ is the angle subtended at the centre, measured in radians.
Period of a function	The period of a function $f(x)$ is the smallest positive number p with the property that $f(x+p) = f(x)$ for all x. The functions $\sin x$ and $\cos x$ both have period 2π and $\tan x$ has period π.
Radian measure	The radian measure θ of an angle in a sector of a circle is defined by $\theta = \frac{\ell}{r}$, where r is the radius and ℓ is the arc length. Thus, an angle whose degree measure is 180 has radian measure π.
Sine rule and cosine rule	The lengths of the sides of a triangle are related to the sine of its angles by the equations $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. This is known as the sine rule.  The lengths of the sides of a triangle are related to the cosine of one of its angles by the equation $c^2 = a^2 + b^2 - 2ab\cos C$. This is known as the cosine rule. 

Sine, cosine and tangent functions	Since each angle θ measured anticlockwise from the positive x-axis determines a point P on the unit circle, we will define the cosine of θ to be the x-coordinate of the point P the sine of θ to be the y-coordinate of the point P the tangent of θ is the gradient of the line segment OP. 
Counting and probability	
Conditional probability	The probability that an event A occurs can change if it becomes known that another event B occurs. The new probability is known as a conditional probability and is written as $P(A\|B)$. If B has occurred, the sample space is reduced by discarding all outcomes that are not in the event B. The new sample space, called the reduced sample space, is B. The conditional probability of event A is given by $P(A\|B) = \frac{P(A \cap B)}{P(B)}$.
Independent events	Two events are independent if knowing that one occurs tells us nothing about the other. The concept can be defined formally using probabilities in various ways: events A and B are independent if $P(A \cap B) = P(A)P(B)$, if $P(A\|B) = P(A)$ or if $P(B) = P(B\|A)$. For events A and B with non-zero probabilities, any one of these equations implies any other.
Mutually exclusive	Two events are mutually exclusive if there is no outcome in which both events occur.
Pascal's triangle	Pascal's triangle is a triangular arrangement of binomial coefficients. The n^{th} row consists of the binomial coefficients $\binom{n}{r}$, for $0 \leq r \leq n$, each interior entry is the sum of the two entries above it, and sum of the entries in the n^{th} row is 2^n 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1 1 6 15 20 15 6 1 1 7 21 35 35 21 7 1 For example, 10 = 4 + 6.
Relative frequency	If an event E occurs r times when a chance experiment is repeated n times, the relative frequency of E is $\frac{r}{n}$.

Unit 2	
Exponential functions	
Algebraic properties of exponential functions	The algebraic properties of exponential functions are the index laws: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, for any real numbers x, y, and a, with $a > 0$.
Index laws	The index laws are the rules: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, and $(ab)^x = a^x b^x$, for any real numbers x, y, a and b, with $a > 0$ and $b > 0$.
Arithmetic and geometric sequences and series	
Arithmetic sequence	An arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant. For instance, the sequence 2, 5, 8, 11, 14, 17, … is an arithmetic sequence with common difference 3. If the initial term of an arithmetic sequence is a and the common difference of successive members is d, then the n^{th} term t_n of the sequence, is given by: $t_n = a + (n-1)d$ for $n \geq 1$. A recursive definition is $t_1 = a, t_{n+1} = t_n + d$, where d is the common difference and $n \geq 1$.
Geometric sequence	A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number called the common ratio. For example, the sequence 3, 6, 12, 24, ... is a geometric sequence with common ratio 2. Similarly the sequence 40, 20, 10, 5, 2.5, … is a geometric sequence with common ratio $\frac{1}{2}$. If the initial term of a geometric sequence is a and the common ratio of successive members is r, then the n^{th} term t_n of the sequence, is given by: $t_n = ar^{n-1}$ for $n \geq 1$. A recursive definition is $t_1 = a, t_{n+1} = rt_n$ for $n \geq 1$ and where r is the constant ratio.
Partial sums of a geometric sequence (geometric series)	The partial sum S_n of the first n terms of a geometric sequence with first term a and common ratio r, $a, ar, ar^2, \dots \dots ar^{n-1}$ … is $S_n = \frac{a(r^n-1)}{r-1}, r \neq 1$. The partial sums form a sequence with $S_{n+1} = S_n + t_n$ and $S_1 = t_1$

Partial sum of an arithmetic sequence (arithmetic series)	The partial sum S_n of the first n terms of an arithmetic sequence with first term a and common difference d. $a, a + d, a + 2d, \ldots\ldots, a + (n-1)d, \ldots\ldots$ is $S_n = \frac{n}{2}(a + t_n) = \frac{n}{2}(2a + (n-1)d)$ where $t_n$is the n^{th} term of the sequence. The partial sums form a sequence with $S_{n+1} = S_n + t_n$ and $S_1 = t_1$
Partial sums of a sequence (series)	The sequence of partial sums of a sequence $t_{1,\ldots,}t_{s\ldots}$is defined by $S_n = t_1 + \ldots + t_n$
Introduction to differential calculus	
Anti-differentiation	An anti-derivative, primitive or indefinite integral of a function $f(x)$ is a function $F(x)$ whose derivative is $f(x)$, i.e. $F'(x) = f(x)$. The process of solving for anti-derivatives is called anti-differentiation. Anti-derivatives are not unique. If $F(x)$ is an anti-derivative of $f(x)$, then so too is the function $F(x) + c$ where c is any number. We write $\int f(x)\,dx = F(x) + c$ to denote the set of all anti-derivatives of $f(x)$. The number c is called the constant of integration. For example, since $\frac{d}{dx}(x^3) = 3x^2$, we can write $\int 3x^2\,dx = x^3 + c$.
Gradient (Slope)	The gradient of the straight line passing through points (x_1, y_1) and (x_2, y_2) is the ratio $\frac{y_2 - y_1}{x_2 - x_1}$. Slope is a synonym for gradient.
Linearity property of the derivative	The linearity property of the derivative is summarised by the equations: $\frac{d}{dx}(ky) = k\frac{dy}{dx}$ for any constant k and $\frac{d}{dx}(y_1 + y_2) = \frac{dy_1}{dx} + \frac{dy_2}{dx}$.
Local and global maximum and minimum	A stationary point on the graph $y = f(x)$ of a differentiable function is a point where $f'(x) = 0$. We say that $f(x_0)$ is a local maximum of the function $f(x)$ if $f(x) \leq f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a global maximum of the function $f(x)$ if $f(x) \leq f(x_0)$ for all values of x in the domain of f. We say that $f(x_0)$ is a local minimum of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a global minimum of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x in the domain of f.
Secant	A secant of the graph of a function is the straight line passing through two points on the graph. The line segment between the two points is called a chord.
Simple polynomial	A simple polynomial is one which is easily factorised and whose stationary points may be easily determined using traditional calculus techniques.
Tangent line	The tangent line (or simply the tangent) to a curve at a given point P can be described intuitively as the straight line that "just touches" the curve at that point. At P where the curve meets the tangent, the curve has "the same direction" as the tangent line. In this sense, it is the best straight-line approximation to the curve at the point P.