Theoretical and Empirical Support for

SAXON MATH™

May 2006

The New Look of Results

Theoretical and Empirical Support for

SAXON MATH™

Table of Contents

Theoretical and Empirical Support for *Saxon Math*

Foundational Research

A well-articulated curriculum challenges students to learn increasingly more sophisticated mathematical ideas as they continue their studies. John Saxon, founder of Saxon Publishers, had a similar philosophy in mind when in the early 1980s he developed his theory-based distributed approach to mathematics instruction, practice, and assessment. Saxon's approach has evolved to include a K–12 textbook series with a comprehensive approach to mathematics.

Because smaller pieces of information are easier to teach and easier to learn, the *Saxon Math* series was developed by breaking down complex concepts into related increments. The instruction, practice, and assessment of those increments were systematically distributed across each grade level. Practice is continual, and assessment is cumulative. The Saxon approach differs from most programs in that it distributes instruction, practice, and assessment instead of massing these elements throughout the lessons and school year. In a massed approach, instruction, practice, and assessment of a skill or concept occur within a short period of time and are clustered within a single chapter or unit. In the *Saxon Math* program, as students encounter new increments of instruction, they are also continually reviewing previously introduced math concepts. Frequent assessments of newer and older concepts are encountered throughout the lessons, ensuring that students truly integrate and retain critical math skills.

Theoretical Framework for *Saxon Math*

Saxon's instructional approach to teaching mathematics is supported by Gagne's (1962, 1965) cumulative-learning theory and Anderson's (1983) ACT theory. Gagne's theory of cumulative learning is based on the premise that intellectual skills can be broken down into simpler skills, which can in turn be divided into even simpler skills. Research has shown that intellectual skill objectives are arranged into a pattern that reveals prerequisite relationships among them (Gagne & Briggs, 1974). Thus, lower level skills must be mastered before higher level skills can be in turn mastered. Anderson's ACT theory explains the development of expertise through three stages: cognitive, associative, and autonomous. During the cognitive stage, learners rehearse and memorize facts related to a particular domain or skill that guide them in problem solving. Within the associative stage, learners are able to detect errors and misunderstandings through continual practice and feedback. By the time learners have reached the autonomous stage, they have practiced a skill to the extent that it becomes automated, reducing the amount of working memory needed to perform the skill and leading to expertise with that skill.

Incremental Instruction Distributed Across the Level

Research also suggests there is value in a teaching method that uses small, easily digestible chunks of information within its lessons (Ausubel, 1969; Brophy & Everston, 1976). Studies by Rosenshine and Stevens (1986) and Brophy and Everston demonstrated the importance of using incremental steps when teaching new information. Effective concept development involves incremental skill instruction distributed throughout a school year.

How *Saxon Math* Addresses the Research. In *Saxon Math*, each increment builds on the foundation of earlier increments, leading students to a deeper understanding of mathematical concepts. The incremental instruction of related elements is carefully distributed throughout each grade level, ensuring that students have the opportunity to master each increment before being introduced to the next related one.

Continual Practice Distributed Across the Level

Foundational research has also shown that distributed instruction results in greater student achievement (English, Wellburn, & Killian, 1934) and leads to a higher level of recall (Glenberg, 1979; Hintzman, 1974) than massed instruction. Distributed instruction with incremental practice and review has been found effective at all grade levels in a variety of subjects, including mathematics, science, and reading comprehension (Dempster, 1988; English et al., 1934; Hintzman, 1974; Reynolds & Glasser, 1964). Several research studies have shown that students who are taught with a mathematics curriculum that uses continual practice and review demonstrate greater math achievement and skill acquisition (Good & Grouws, 1979; Hardesty, 1986; MacDonald, 1984; Mayfield & Chase, 2002; Ornstein, 1990; Usnick, 1991). Dempster (1991) noted that the benefits of review have been validated by research since the early part of the 20th century, and numerous studies suggest that when review is incorporated into the learning process both the quantity and quality of what is learned is improved. Studies in cognitive science also support continual practice, because it develops computational automaticity—it increases retrieval speed, reduces time required for recognition, and decreases interference (Klapp, Boches, Trabert, & Logan, 1991; Pirolli & Anderson, 1985; Thorndike, 1921).

How *Saxon Math* Addresses the Research. In *Saxon Math*, practice of an increment is continual and distributed across each grade level. After an increment of a concept is introduced, students are given multiple opportunities and ample time to practice it. This allows students to understand and master the increment before being introduced to a related increment of the concept. Continual, distributed practice ensures that concepts are committed to students' long-term memory and that students achieve automaticity of basic math skills. The Saxon philosophy holds that all students must acquire basic-skills proficiency before they are able to progress to higher order mathematical thinking.

Effective concept development involves incremental skill instruction, distributed throughout a school year.

Cumulative Assessment Distributed Across the Level

In terms of cumulative assessment, research has indicated that well-designed classroom testing programs that are routine rather than an interruption (NCTM, 2000) have a positive impact on later student achievement (Dempster, 1991). Dempster found that higher levels of achievement occur when testing is frequent and cumulative rather than infrequent or related only to content covered since the last test. Benefits are most noted when tests are an integral part of the instructional approach; administered regularly and frequently; and collected, scored, recorded, and returned to students promptly, thus preventing any misunderstanding from becoming ingrained. Furthermore, Cotton (2001) noted that students who are tested frequently and given feedback have more positive attitudes toward tests.

According to Fuchs (1995), assessments enhance instruction by monitoring student learning, evaluating instructional programs, and revealing remediation needs. In particular, cumulative assessment that is frequent and distributed has been found to be effective by a number of studies that have shown that students who are assessed frequently have higher test scores than students who are assessed infrequently (Blair, 2000; Peckham & Row, 1977; Rohm, Sparzo, & Bennett, 1986).

How *Saxon Math* Addresses the Research. The frequent, cumulative assessments in *Saxon Math* examine both the acquisition and maintenance of concepts. Assessments are provided at regular intervals to help teachers frequently gauge students' progress. Furthermore, because each assessment is cumulative, teachers can also monitor students' retention of skills. Frequent, cumulative assessment is a natural complement to Saxon's distributed approach to incremental instruction and continual practice. In the *Saxon Math* program, assessments are given on a weekly basis and cover mathematical concepts that have been previously taught.

Efficacy Studies

Historical Effectiveness of *Saxon Math*: Elementary & Middle School

A number of scientific studies have demonstrated the instructional effectiveness of *Saxon Math* at grades K–8. In 2005, Harcourt Achieve contracted with PRES Associates—an external, independent educational research firm—to conduct analyses using archival state assessment data on the effectiveness of their Saxon elementary and middle school math programs in the state of Georgia in grades 1st–8th (PRES Associates, December 2005) and a second separate analysis in the state of Texas in grades 6th–8th grade (PRES Associates, April 2005).[1] Both analyses were conducted on school-level achievement data from schools using the *Saxon Math* during specified years and those schools that used other math curricula during the same years.

---

[1] For full copies of these research reports, please visit the Saxon Web site at www.harcourtachieve.com

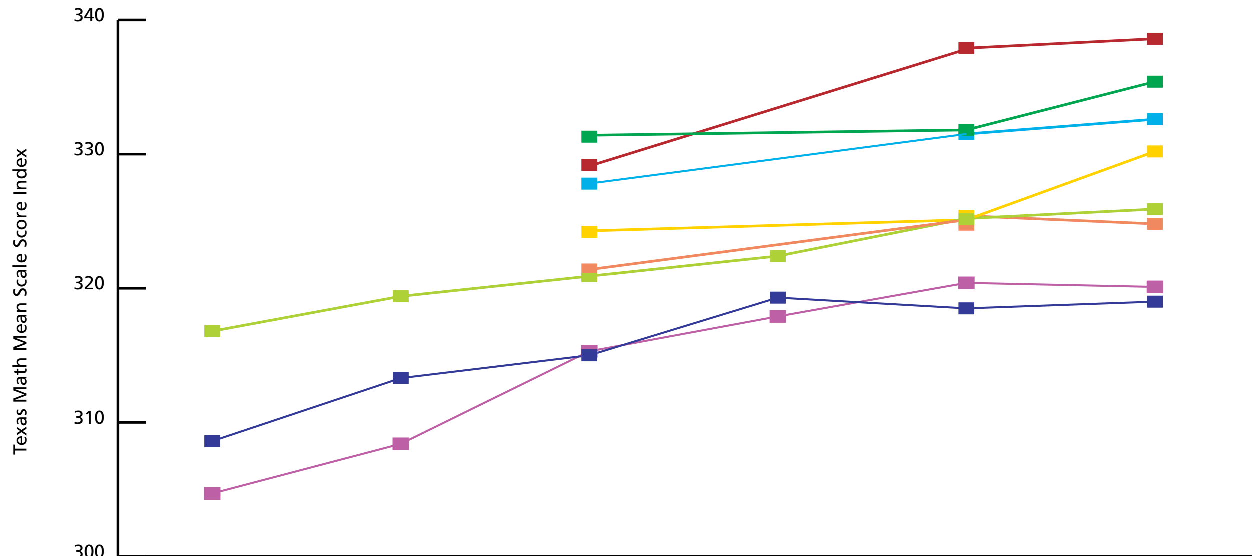
Georgia Elementary and Middle Schools

Major findings of the study conducted in Georgia indicated that among Saxon schools there was significant growth in math performance within all grade levels (1st–8th) from spring 2000 to spring 2005 (see Figure 1). Additionally, growth among Saxon schools on the Georgia Criterion Referenced Competency Test (CRCT) for math was not dependent on how long a school had used the program. Therefore, schools that had only implemented the Saxon program for 1 year showed the same rates of growth as schools that had implemented the program for 5 or more years. Furthermore, schools that used *Saxon Math* programs showed an improvement in math performance after just one year after exposure to Saxon as compared to their performance prior to Saxon and to schools that did not use Saxon that year. Overall, analyses of longitudinal data over the past 6 years in Georgia showed that the Saxon elementary and middle school math program were associated with positive and significant outcomes in math achievement, as measured by the Georgia statewide assessment (CRCT).

A number of scientific studies have demonstrated the instructional effectiveness of* Saxon Math *at grades K–8.

Figure 1.

Saxon schools' math performance growth (1st–8th) on CRCT math scale score

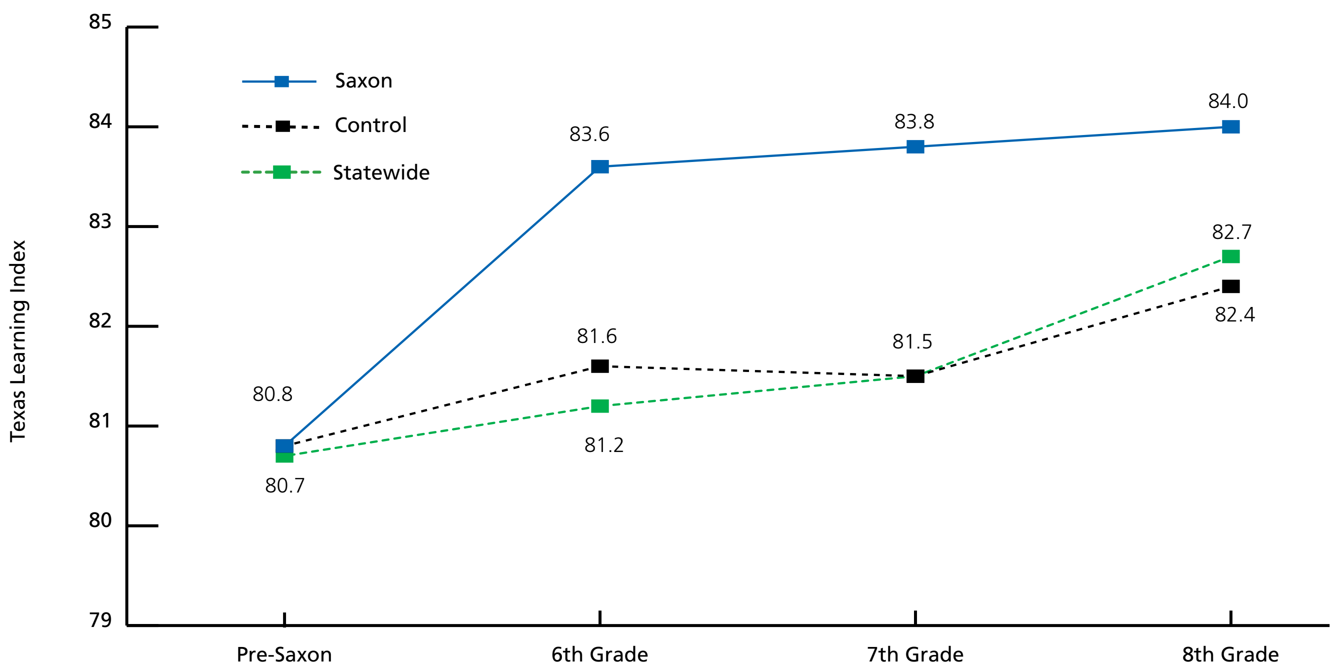


	2000	2001	2002	2003	2004	2005
Grade 1			329.2		337.9	338.6
Grade 2			331.3		331.8	335.4
Grade 3			327.8		331.5	332.6
Grade 4	308.6	313.3	315.0	319.3	318.5	319.0
Grade 5			324.2		325.1	330.2
Grade 6	316.8	319.4	320.9	322.4	325.2	325.9
Grade 7			321.0		325.4	324.8
Grade 8	304.7	308.4	315.3	317.9	320.4	320.1

Texas Middle Schools

The results of the study conducted on archival data from Texas middle schools demonstrated that *Saxon Math* students showed significant growth on the *Texas Assessment of Academic Skills* (TAAS) from 6th to 8th grade (1998–2001). This growth was not dependent upon how long a school had used the program prior to 1998. Students in 6th–8th grade using *Saxon Math* programs outperformed students who attended these schools prior to implementing the Saxon programs. Furthermore, an analysis of a matched sample of Saxon and non-Saxon schools found that, in all three grade levels, Saxon students outperformed non-Saxon students and scored higher than the statewide average after implementing *Saxon Math* (see Figure 2). More recent data from these schools found that across all three grade levels, a higher proportion of Saxon students passed the *Texas Assessment of Knowledge and Skills* (TAKS) math standard and performed better on all math objectives measured by the TAKS compared to non-Saxon students and the statewide average. Most significantly, the analysis found that a higher percentage of students who used *Saxon Math* in grades 6 – 8 met the TAKS minimum requirements on the exit-level (10th grade) test compared to non-Saxon students.

Figure 2.
Growth in TAAS Texas Learning Index by group (and statewide)



Experimental Studies: Kindergarten – Eighth Grade

A number of experimental and quasi-experimental evaluations of the *Saxon Math* program (K–8) have also been conducted through independent research organizations including universities and school district evaluation departments.[2]

Kindergarten – Fifth Grade

Two large-scale, quasi-experimental studies on the effectiveness of *Saxon Math* were conducted during two school years (1992–1994) by the Planning, Research, and Evaluation Department of Oklahoma City Public Schools (Nguyen, 1994; Nguyen & Elam, 1993). During the first year of the evaluation, 1992–1993, researchers from the Oklahoma City Public Schools Research, Planning, and Evaluation Department examined student achievement from five Oklahoma City schools that had fully implemented the *Saxon Math* program in Kindergarten through fifth grade (Nguyen & Elam, 1993). These five schools had been implementing *Saxon Math* for two years at the time of data collection, minimizing any negative effects due to implementing a newly acquired program.

Achievement on the math subtests of the *Iowa Tests of Basic Skills* (ITBS) for students at the Saxon schools was compared to achievement from a matched-sample of students selected to be the control group who were in classrooms that were using a Scott Foresman math text. Students were matched on grade level, gender, race, socio-economic status (SES), and the year prior ITBS total math score. In general, students using the *Saxon Math* program scored significantly higher than the control group on five out of the nine subtests of the ITBS: Complete Composite, Total Mathematics, Mathematics Concepts, Problem Solving, and Reading Comprehension ($p < .05$ for all significant comparisons).

Student achievement was also examined by grade level. Grades 3, 4, and 5 from the five Saxon schools and a matched control sample from non-Saxon schools were chosen for comparison. Saxon students had higher achievement on 23 out of the 27 grade-level comparisons on the ITBS subtests. Eleven of these differences were significant in favor of the Saxon group ($p < .05$). A specific pattern of results within grade levels was not found, but generally the Saxon group outperformed the control group on the majority of the grade-level subtest comparisons.

In order to further evaluate the effectiveness of *Saxon Math* at the elementary level, a second study was conducted by the Oklahoma City Public Schools, Planning, Research, and Evaluation Department to examine student achievement in math in Oklahoma City schools (Nguyen, 1994).This

The Saxon approach differs from most programs in that it distributes instruction, practice, and assessment… throughout the lessons and school year.

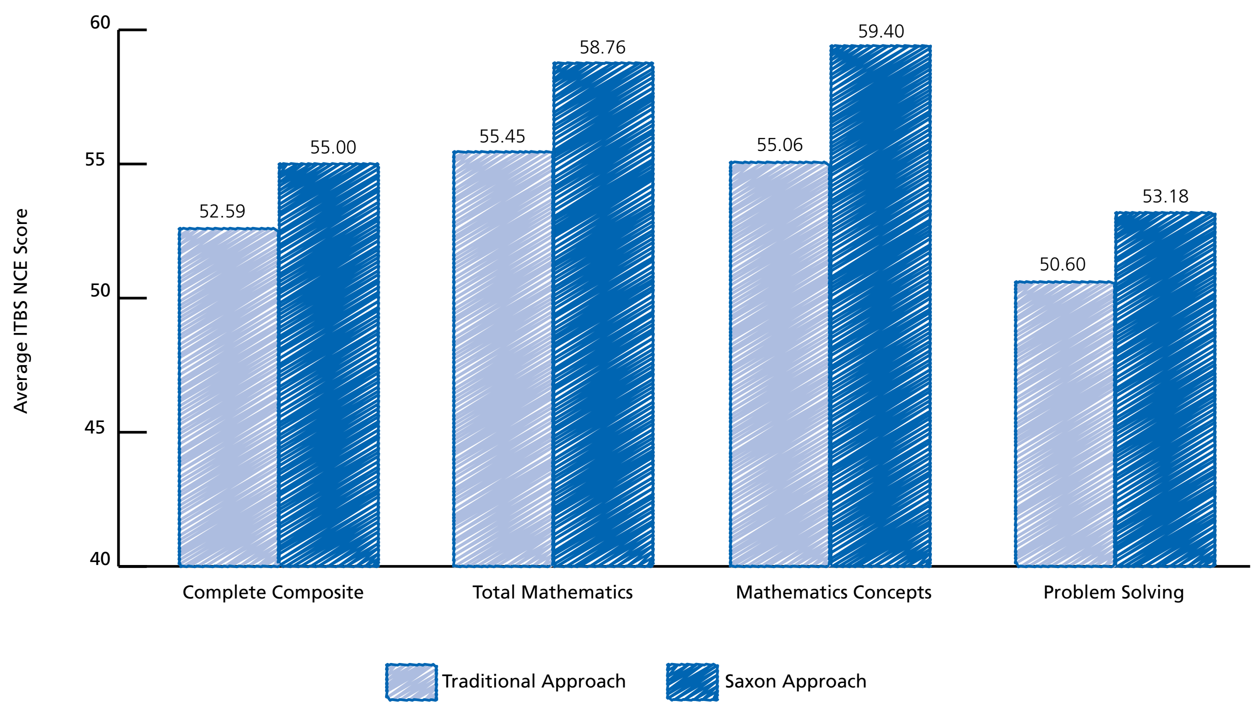
[2] A report summarizing the following independent evaluations of *Saxon Math* can be obtained online from the Saxon Web site at www.harcourtachieve.com

study was conducted with five elementary schools in Oklahoma City that had completely integrated the Saxon program in all grade levels. A matched sample of the students using the Scott Foresman math text was selected to be the control group. These students were matched to students using the Saxon program on grade level, gender, race, SES, and the year prior ITBS total math score.

The ITBS scores from the 1993–1994 school year were collected and used to evaluate growth in mathematic skill over the implementation time for both groups.

On the posttest ITBS, the students using the *Saxon Math* product outscored the control group students on all subtests: Complete Composite, Total Math, Problem Solving, Reading Comprehension, Math Computation, Math Concepts, Science, and Social Studies. However, only the differences between groups on the Math Concepts, Science, and Social Studies tests were significant ($p < .05$). These results indicate that, collapsed across grade levels, students who used *Saxon Math* at these Oklahoma schools achieved greater gains in their knowledge of math concepts than did students using the Scott Foresman program (see Figure 3).

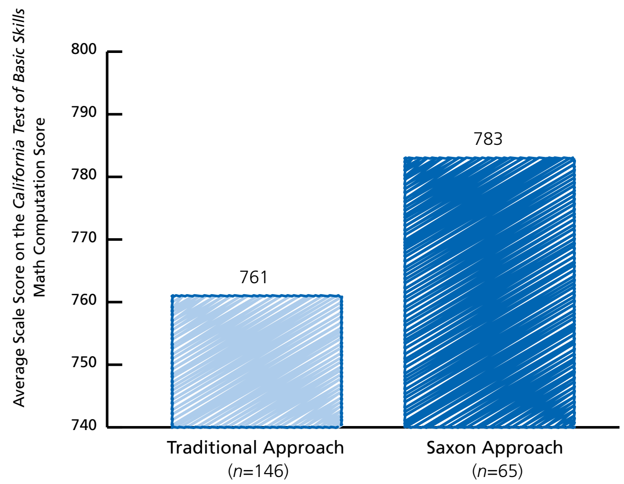
Figure 3.
Iowa Test of Basic Skills Math Subtest Comparisons



Sixth Grade

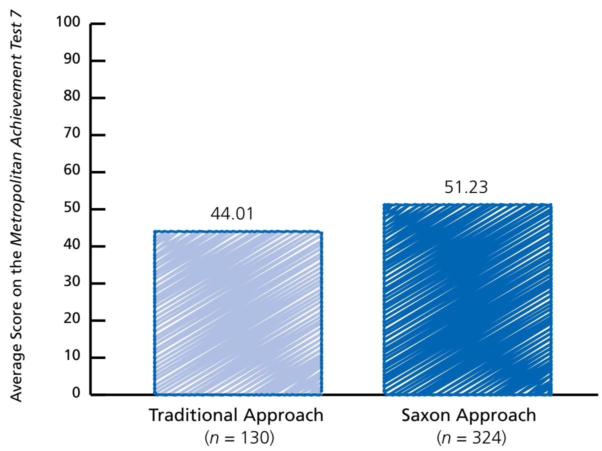
Several examinations of the effectiveness of *Saxon Math* at the sixth-grade level have found superior performance for the Saxon approach than programs based on the traditional unit-based approach. Specifically, Rentschler (1994) found that after controlling for pretest differences, Saxon students significantly outperformed a matched-sample of students using a traditional unit-based program on the mathematics computation subtest of the *California Test of Basic Skills* (see Figure 4). Similarly, Lafferty (1994) reported that sixth-grade students using the Saxon program scored significantly higher scores on the MAT 7 than students in traditional-approach classrooms after controlling for pretest differences (see Figure 5). Furthermore Lafferty found that the Saxon students had significantly less math anxiety at the end of the year than the students in traditional approach classrooms.

Figure 4.
Posttest Comparison of Math Computation Ability



Group differences for the adjusted posttest means significantly different at the 95% confidence interval. A post-hoc effect size was calculated from the existing data to be *d* = .73.

Figure 5.
Mathematics Acievement Posttest Comparison for Sixth-Grade Students Using *Saxon Math* Versus a Control-Group Math Product



Posttest mean differences between the two groups were significant at the 95% confidence interval. A post-hoc effect size was calculated from the existing data to be *d* = .42

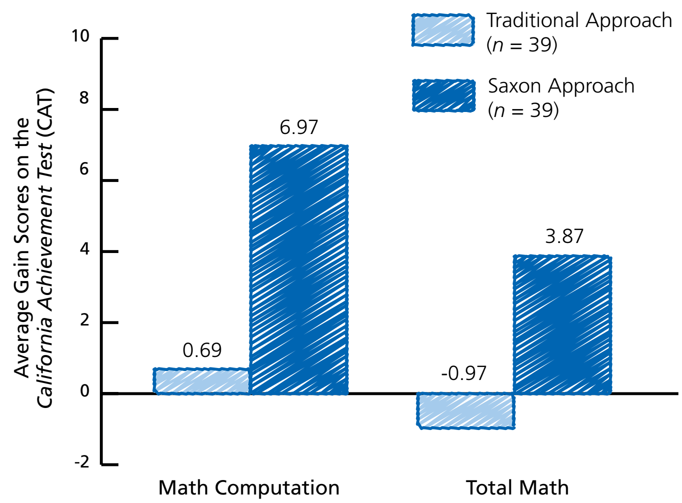
Eighth Grade

Multiple investigations of the Saxon methodology have also be conducted at the eighth-grade level to examine the effectiveness of the *Saxon Algebra I* and *Algebra ½* programs. Results of a 9-week study of *Algebra I* conducted by Clay (1998) found that *Saxon Math* was effective at increasing math achievement scores on teacher-created criterion referenced math tests and helped students overcome an initial math deficiency to bring about greater gains in math achievement than a control group using a traditional program.

Crawford and Raia (1986) examined the effectiveness of *Saxon Algebra ½* with eighth grade students in five different middle schools. Achievement on the *California Achievement Test* (CAT) for students in Saxon classrooms was compared to those students in classrooms using a more traditional approach to math instruction. After controlling for pretest differences, it was found that the results significantly favored the Saxon program. A second analysis matching students in the treatment and control groups by preimplementation achievement level found that Saxon students made significantly higher gains from pre–post on the CAT Math Computation subtest and the Total Math score than control group students (see Figure 6). A final analysis examined achievement on only those objectives covered by both programs. The analysis found that the results were significantly different in favor of the Saxon group indicating that the higher gains in achievement on the CAT for the *Saxon Math* students were not due to performance on objectives that were unique to the Saxon program.

Finally, in a recent evaluation of the *Saxon Math* program, Baldree (2003) found that eighth-grade students who used the *Saxon Math* program had significantly higher scores on the Computation and Concepts and Estimation subtests of the *Georgia Criterion Referenced Competency Test* than a matched group of students who received Pre-Algebra instruction based on a constructivist-based model. Taken together, all these results provide a strong body of evidence to support the instructional effectiveness of the *Saxon Math* programs from Kindergarten to eighth grade.

Figure 6.
Gain Scores on the *California Achievement Test: Math*



The average gain for the Saxon group on Math Computation and Total Math score was significantly higher than the Control group at the 95% confidence interval. Post-hoc effect sizes were calculated from the existing data to be *d* = .55 for Math Computation and *d* = .62 for Total Math.

References

Anderson, J. (1983). *The architecture of cognition.* Cambridge, MA: Harvard University Press.

Ausubel, D. P. (1969). *Readings in school learning.* New York: Holt, Rinehart, and Winston.

Baldree, C. L. P. (2003). *The effectiveness of two mathematical instructional programs on the mathematics growth of eighth grade students.* Unpublished doctoral dissertation, University of Georgia, Athens, Georgia.

Blair, J. (2000). ETS study links effective teaching methods to test-score gains. *Education Week, 20*(8), 24.

Brophy, J., & Everston, C. (1976). *Learning from teaching: A developmental perspective.* Boston: Allyn and Bacon.

Clay, D. W. (1998). *A study to determine the effects of a non-traditional approach to algebra instruction on student achievement.* Unpublished master's thesis, Salem-Teikyo University.

Cotton, K. (2001). *Monitoring student learning in the classroom.* Northwest Regional Educational Laboratory. Retrieved October 8, 2002, from http://www.nwrel.org/scpd/sirs/2/cu4.html

Crawford, J. R., & Raia, F. (1986). *Executive summary: Analysis of eighth grade math texts and achievement.* Oklahoma City: Oklahoma City Public Schools, Planning, Research, and Evaluation Department.

Dempster, F. (1988). The spacing effect: A case study in the failure to apply results to psychological research. *American Psychologist, 43,* 627–634.

Dempster, F. (1991, April). Synthesis of research on reviews and tests. *Educational Leadership, 48,* 71–76.

English, H. B., Wellburn, E. L., & Killian, C. D. (1934). Studies in substance memorization. *Journal of General Psychology, 11,* 233–260.

Fuchs, L. S. (1995). *Connecting performance assessment to instruction: A comparison of behavioral assessment, mastery learning, curriculum-based measurement, and performance assessment.* (ERIC Document Reproduction Service No. ED381984).

Gagne, R. M. (1962). The acquisition of knowledge. *Psychological Review, 69,* 355–365.

...all these results provide a strong body of evidence to support the instructional effectiveness of the* Saxon Math *programs.

Gagne, R. M. (1965). *The conditions of learning.* Austin, TX: Holt, Rinehart, and Winston.

Gagne, R. M., & Briggs, L.J. (1974). *Principles of instructional design.* Fort Worth, TX. HBJ College Publisher.

Glenberg, A. M. (1979). Component-levels theory of the effects of spacing of repetitions on recall and recognition. *Memory and Cognition, 7,* 95–112.

Good, T. L., & Grouws, D.A. (1979). The Missouri mathematics effectiveness project. *Journal of Educational Psychology, 71,* 355–362.

Hardesty, B. (1986). Notes and asides. *National Review, 37,* 21–22.

Hintzman, D. L. (1974). Increasing your teaching effectiveness. In R.L. Solso (Ed.), *Theories in cognitive psychology: The Loyola symposium* (pp. 77–99). Potomac, MD: Erlbaum.

Klapp, S. T., Boches, C. A., Trabert, M. L., & Logan, G. D. (1991). Automatizing alphabet arithmetic: II. Are there practice effects after automaticity is achieved? *Journal of Experimental Psychology: Learning, Memory, and Cognition, 17,* 196–209.

Lafferty, J. F. (1994). *The links among mathematics text, students' achievement, and students' mathematics anxiety: A comparison of incremental development and traditional texts.* Unpublished doctoral dissertation, Widener University, Chester, Pennsylvania.

MacDonald, C. J. (1984). A comparison of three methods of utilizing homework in a precalculus college algebra course. *Dissertation Abstracts International, 45,* 164A.

Mayfield, K. H., & Chase, P. N. (2002). The effects of cumulative practice on mathematics problem solving. *Journal of Applied Behavior Analysis, 35,* 105–123.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Nguyen, K. (1994). *The 1993–94 Saxon mathematics program evaluation report.* Oklahoma City: Oklahoma City Public Schools.

Nguyen, K., & Elam, P. (1993). *The 1992–93 Saxon mathematics program evaluation report.* Oklahoma City: Oklahoma City Public Schools.

Ornstein, A. C. (1990). Practice and drill: Implications for instruction. *National Association of Secondary School Principals, 74,* 112–117.

Peckham, P. D., & Roe, M. D. (1977). The effects of frequent testing. *Journal of Research and Development in Education, 10*(3), 40–50.

Pirolli, P. L., & Anderson, J. R. (1985). The role of practice in fact retrieval. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 11,* 136–153.

PRES Associates. (2005, April). *The relationship between using* Saxon Middle School Math *and student performance on Texas statewide assessments.* Austin, TX: Author.

PRES Associates. (2005, December). *The relationship between using* Saxon Elementary and Middle School Math *and student performance on Georgia statewide assessments.* Austin, TX: Author.

Rentschler, R. V. (1994). *The effects of Saxon's incremental review on computational skills and problem-solving achievement of sixth-grade students.* Dissertation Abstracts International, 56(02), 484A. (UMI No. 9518017).

Reynolds, J. H., & Glasser, R. (1964). Effects of repetition and spaced review upon the retention of a complex learning task. *Journal of Educational Psychology, 55,* 297–308.

Rohm, R. A., Sparzo, F. J., & Bennett, C. M. (1986). College student performance under repeated testing and cumulative conditions: Report on five studies. *Journal of Educational Research, 80*(2), 99–104.

Rosenshine, B., & Stevens, R. (1986). Teaching functions. In M. C. Wittrock (Ed.), *Handbook of research on teaching: Vol. 3.* (pp. 376–391). New York: Macmillan.

Thorndike, E. L. (1921). The psychology of drill in arithmetic: The amount of practice. *The Journal of Educational Psychology, 12,* 183–194.

Usnick, V. F. (1991). It's not drill and practice, it's drill or practice. *School Science and Mathematics, 91,* 344–347.

SAXON™

A Harcourt Achieve Imprint

www.SaxonMath.com
1.800.284.7019

33.5M/NY/SPECTRA/06-06
9994038125 IND
9994038133 10PK