Comments on the Steinhart-Hart Equation

Application Note

Nov 11, 2015

In 1968 Dr. John S. Steinhart and Dr. Stanley R. Hart, both researchers at Carnegie Mellon University, wrote a paper that defined the industry standard equation for a thermistor's temperature versus resistance transfer function. The classic Steinhart and Hart equation has the form:

$$1/T = A_0 + A_1(\ln R) + A_3(\ln R)^3$$

Where: T = Temperature in Kelvins (Kelvin = Celsius + 273.15)

A_0, A_1, A_3 = Constants derived from thermistor measurements

R = Thermistor's resistance in Ohms

ln = Natural Log (Log to the Napierian base 2.718281828…)

In practice, three thermistor resistance measurements are made at three defined temperatures. These temperatures are usually the two endpoints and the center point of the temperature range of interest. The equation hits these three points directly and has a small error over the range.

Why does the Steinhart-Hart equation have the form that it has?

During the mid 1960's when the equation was developed, engineering computations were performed by hand with the help of mechanical desktop calculators, slide rules and transcendental function table books. Most engineers never had access to the mainframe computers of the day. Steinhart and Hart had to define an equation simple enough to do by hand.

If you read Steinhart and Hart's paper the equation actually has the form of;

$$1/T = A_0 + A_1(\ln R) + A_2(\ln R)^2 + A_3(\ln R)^3 + A_4(\ln R)^4 + A_5(\ln R)^5 \ldots$$

Added terms can go to infinity. Because the error is small and to save computation time, the formula was truncated after the cubed term and the squared term is dropped because its constant is very, very small.

Today our PCs allow us to do a better job. Modern spread sheet programs derive the equation constants through polynomial regression based on a least squares approximation. Instead of three values, hundreds may be used to figure out the constants. The resultant constants are picked for the least total error across the temperature range of interest. Least squares approximation has been known for a long time; in 1967 it would have taken several months of 8hr/day hand calculations to figure out the constants.

Steinhart and Hart coefficients were solved using polynomial regression for the temperature range 0°C to 70°C for the 10K-2 thermistor. Figure 1, on the next page, shows the Steinhart-Hart equation error for the "classic" and full 3rd order polynomial solutions. As shown, the full third order polynomial has smaller and more symmetrical error.

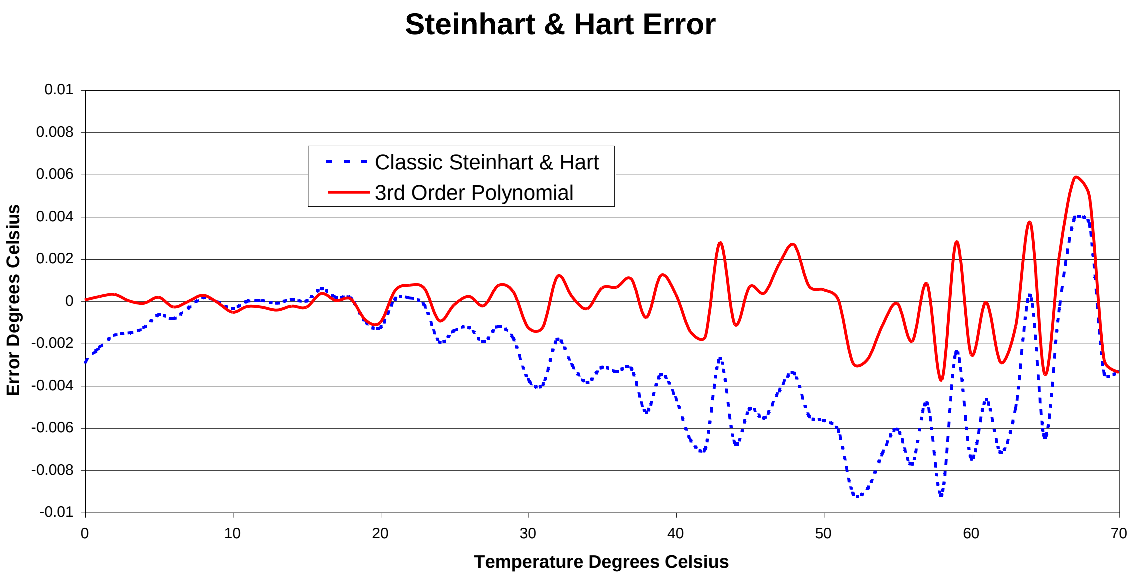

Figure 1: Steinhart and Hart equation error

The classic equation finds temperature from thermistor resistance. Many times one needs to compute the thermistor resistance when the temperature is known. The classic equation can be manipulated to find resistance and it looks like this;

$$R = \exp\left[\sqrt[3]{-\left[\frac{\frac{(A-T^{-1})}{C}}{2}\right]+\sqrt[2]{\left[\frac{\left[\frac{(A-T^{-1})}{C}\right]^2}{4}+\frac{\left(\frac{B}{C}\right)^3}{27}\right]}}+\sqrt[3]{-\left[\frac{\frac{(A-T^{-1})}{C}}{2}\right]-\sqrt[2]{\left[\frac{\left[\frac{(A-T^{-1})}{C}\right]^2}{4}+\frac{\left(\frac{B}{C}\right)^3}{27}\right]}}\right]$$

This is a complicated and messy equation. It does have the advantage that it uses the same constants as the temperature equation.

BAPI engineers independently developed an easier equation to find resistance, but it requires different constants than those used for the temperature equation. Others have developed the identical equation and it is published on several websites.

$$\ln(R) = B_0 + B_1(1/T) + B_2(1/T)^2 + B_3(1/T)^3$$
$$R = e^{\ln(R)}$$

For the 10K-2 thermistor the "B" constants over the 0°C to 70°C temperature range are;

B_0 = -5.380125
B_1 = 4,777.517
B_2 = -120,014.68
B_3 = -2,168,775

The error in Ohms is shown in Figure 2.

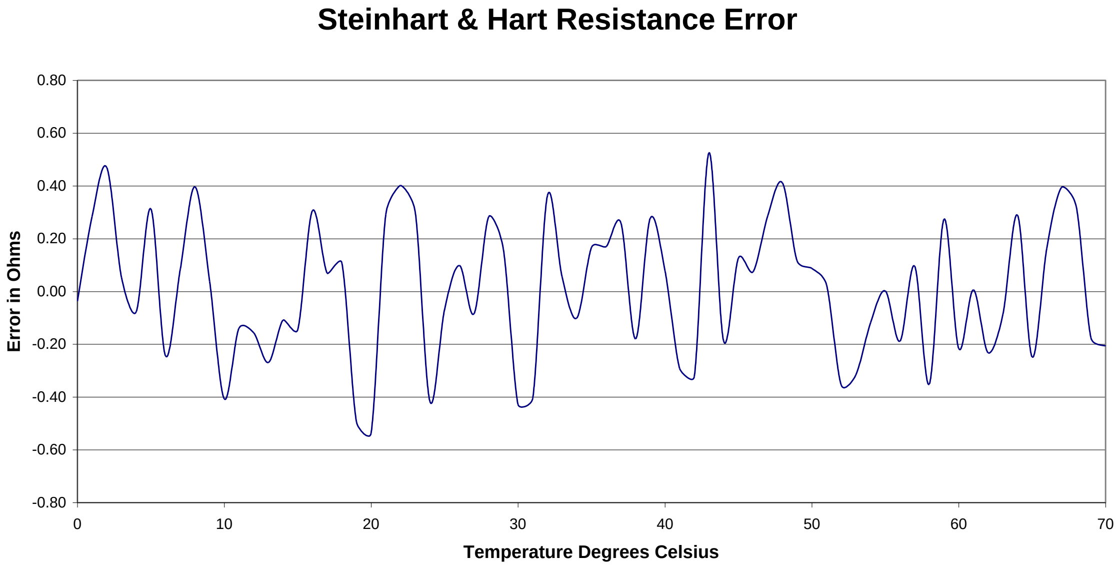

Figure 2: Error for the Steinhart & Hart resistance third order polynomial equation

Updating the Steinhart and Hart equation constants to modern computational methods allows for simpler equations that can be solved using industry standard spreadsheet programs with less error.

The following page has tables of A and B coefficients for all of BAPI's popular thermistors over the temperature range of 0°C to 70°C or 32°F to 158°F.

References

John S. Steinhart, Stanley R. Hart, Calibration curves for thermistors, Deep Sea Research and Oceanographic Abstracts, Volume 15, Issue 4, August 1968, Pages 497-503, ISSN 0011-7471, DOI: 10.1016/0011-7471(68)90057-0.

1.8K Thermistor	
A_0 = 1.285927E-03	B_0 = -5.333625E+00
A_1 = 2.590738E-04	B_1 = 4.290757E+03
A_2 = 1.307688E-06	B_2 = -1.318809E+05
A_3 = 1.251364E-07	B_3 = -2.078997E+06

3K Thermistor	
A_0 = 1.404029E-03	B_0 = -6.160373E+00
A_1 = 2.372439E-04	B_1 = 4.389885E+03
A_2 = -3.280711E-08	B_2 = -2.383189E+03
A_3 = 1.025105E-07	B_3 = -1.405035E+07

3.25K Thermistor	
A_0 = -3.373240E-03	B_0 = 1.187211E+01
A_1 = 2.233328E-03	B_1 = -1.399569E+04
A_2 = -2.833486E-04	B_2 = 6.393488E+06
A_3 = 1.360832E-05	B_3 = -7.624648E+08

3.3K Thermistor	
A_0 = 1.405066E-03	B_0 = -6.487879E+00
A_1 = 2.279989E-04	B_1 = 4.777517E+03
A_2 = 1.121519E-06	B_2 = -1.201468E+05
A_3 = 5.252617E-08	B_3 = -2.168775E+06

10K-2 Thermistor	
A_0 = 1.153805E-03	B_0 = -5.380125E+00
A_1 = 2.257075E-04	B_1 = 4.777517E+03
A_2 = 9.469611E-07	B_2 = -1.201468E+05
A_3 = 5.252617E-08	B_3 = -2.168775E+06

10K-3 Thermistor	
A_0 = 1.061244E-03	B_0 = -5.121667E+00
A_1 = 2.275747E-04	B_1 = 5.001281E+03
A_2 = 1.371993E-06	B_2 = -2.287593E+05
A_3 = 1.029782E-07	B_3 = 3.462198E+06

10K-3[11K] Thermistor	
A_0 = -5.685951E-02	B_0 = -1.132408E+01
A_1 = 2.239259E-02	B_1 = 8.544361E+03
A_2 = -2.831535E-03	B_2 = -3.549525E+05
A_3 = 1.211816E-04	B_3 = -1.266395E+08

10K-4 Thermistor	
A_0 = 8.345089E-04	B_0 = -3.858074E+00
A_1 = 2.696376E-04	B_1 = 4.380393E+03
A_2 = -2.069132E-06	B_2 = -1.192215E+05
A_3 = 2.707289E-07	B_3 = -7.491021E+06

20K Thermistor	
A_0 = 1.132230E-03	B_0 = -5.637730E+00
A_1 = 2.191526E-04	B_1 = 4.924457E+03
A_2 = -6.081209E-07	B_2 = -3.038523E+04
A_3 = 1.143152E-07	B_3 = -1.679267E+07

30K Thermistor	
A_0 = 1.092058E-03	B_0 = -5.795494E+00
A_1 = 2.058109E-04	B_1 = 5.419606E+03
A_2 = 5.485526E-07	B_2 = -1.742797E+05
A_3 = 7.474987E-08	B_3 = -2.972593E+06

50K Thermistor	
A_0 = 9.308236E-04	B_0 = -4.721440E+00
A_1 = 2.205549E-04	B_1 = 4.924457E+03
A_2 = -9.223586E-07	B_2 = -3.038523E+04
A_3 = 1.143152E-07	B_3 = -1.679267E+07

100K Thermistor	
A_0 = 7.774654E-04	B_0 = -4.028292E+00
A_1 = 2.219984E-04	B_1 = 4.924457E+03
A_2 = -1.160070E-06	B_2 = -3.038523E+04
A_3 = 1.143152E-07	B_3 = -1.679267E+07